Computing Line Integrals

Are we integrating a **scalar function** f or a **vector field** $\vec{F}$?

scalar function →

$$\int_C f\, ds = \int_a^b f(\vec{r}(t))\, \left|\vec{r}\,'(t)\right|\, dt$$

can be used for any scalar function

vector field ↓

Is $\vec{F}$ **conservative** (i.e. is $\vec{F} = \vec{\nabla} f$)?

no →

Is C a simple **closed path** enclosing a region D?

no →

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

can be used for any vector field

yes ↓

Fundamental Theorem:

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{\nabla} f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

yes ↓

Green's Theorem, $\vec{F} = \langle P, Q \rangle$:

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D Q_x - P_y\, dA$$

only for two-dimensional line integrals