Name: _____

Score: _____ /20

Line Integrals

Please staple your work and use this page as a cover page.

1. Evaluate the line integral $\int_C (x^2 + 2y)\, ds$ if C is the line segment from $(0, 3)$ to $(4, 6)$.
2. Evaluate the line integral $\int_C (x^2 + y^2 + z^2)\, ds$ if C is the curve given by $x = t$, $y = \cos 2t$, $z = \sin 2t$, $0 \leqslant t \leqslant 2\pi$.
3. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F}(x, y) = \langle xy, 3y^2 \rangle$ and C is the curve given by $x = 11t^4$, $y = t^3$, $0 \leqslant t \leqslant 1$.
4. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F}(x, y, z) = \langle x, y, xy \rangle$ and C is the curve given by $x = \cos t$, $y = \sin t$, $z = t$, $0 \leqslant t \leqslant \pi$.
5. Use a line integral to prove that the lateral surface area of a cylinder of height h and radius r is $2\pi rh$.
6. Consider the force field $\vec{F}(x, y) = \langle x^2, xy \rangle$.
   (a) If a particle moves once around the circle $x^2 + y^2 = 4$ in the counter-clockwise direction, find the work done by the given force field on the particle.
   (b) Does anything change if the particle moves around the circle in the clockwise direction instead? Explain.
7. Show that a constant force field does zero work on a particle that moves once around the circle $x^2 + y^2 = 1$. Is the work still zero if the radius is not 1? Explain.