Name: _____

Score: _____ /20

Double Integrals and Volume

Please staple your work and use this page as a cover page.

1. This first question is intended to be review and a warm-up activity. Use the results of this problem to help you think about the following questions.
   (a) Find the area of the region contained between $y = x$ and $y = x^2$ if $0 \leqslant x \leqslant 4$.
   (b) Find the area of the region contained between $y = x^3$ and the x-axis if $-2 \leqslant x \leqslant 2$.

2. Double integrals can be used to compute volumes of solids, but just like when finding area of regions with a single integral, there are certain pitfalls that need to be considered.
   (a) Find the volume contained between the surfaces $z = 3x + 2y$ and $z = 0$ over the rectangle $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 2$.
   (b) Find the volume contained between the surfaces $z = -x^2 - y^2$ and $z = 0$ over the same rectangle.
   (c) Use your answers to parts (a) and (b) to write a single iterated integral that could be used to determine the volume between the surfaces $z = 3x + 2y$ and $z = -x^2 - y^2$ over the rectangle $0 \leqslant x \leqslant 1$, $0 \leqslant y \leqslant 2$.
   (d) What is the value of the integral in part (c)? Explain your answer.

3. (a) Find the value of the double integral below if D is the region given by $x^2 + y^2 \leqslant 4$.

$$\iint_D (x - y)\, dA$$

   (b) Say that we wanted to find the volume contained between the planes $z = x - y$ and $z = 0$ inside the cylinder $x^2 + y^2 = 4$. Explain why the integral in part (a) does not yield the volume.
   (c) Sketch a picture to show geometrically why the value of the integral in (a) is 0.
   (d) How could you use a double integral and symmetry to determine the desired volume? Think about the picture that you drew in part (c) and use it to set up and evaluate an integral that will yield the volume.

4. You only need to set up the integrals in parts (a)-(c).
   (a) Say that we want to find the volume contained between $z = xy$ and the xy-plane over the region D bounded by $x = 0$, $y = 0$, $y = 1 - x$, and $y = 2 - 2x$. Sketch a picture of this region. Would it be easier to use Cartesian or polar coordinates to compute this double integral? Decide which is easier and set up the double integral to determine this volume.
   (b) Consider the region D in the first quadrant given by $1 \leqslant x^2 + y^2 \leqslant 4$. Set up the two double integrals that could be used to compute the volume between $z = xy$ and xy-plane over this region D. Why does it take two double integrals and not just one?
   (c) Consider the same region as in part (b). Use polar coordinates to set up the integral instead. This time it will only take one double integral to find the volume. Why?
   (d) Compute the integral from part (c).