Name: ____________________

Score: ________ /20

Using the Divergence Theorem

Please staple your work and use this page as a cover page.

1. Let $\vec{F} = \langle z, y, x\rangle$ and let S be the surface $x^2 + y^2 + z^2 = 16$ with outward orientation.
   (a) Compute the flux of $\vec{F}$ across the surface S using the **definition** of a surface integral.
   Hint: Show that you can parameterize S via $\vec{r}(u, v) = \langle 4\sin u\cos v, 4\sin u\sin v, 4\cos u\rangle$. Once you do that, you may feel free to use the following (without showing work):
   $$\vec{r}_u \times \vec{r}_v = \langle 16\sin^2 u\cos v, 16\sin^2 u\sin v, 16\sin u\cos u\rangle.$$
   (b) Use the Divergence Theorem to find the flux instead.

2. Let S be the surface of the solid bounded by $y^2 + z^2 = 1$, $x = -1$, and $x = 2$ and let $\vec{F} = \langle 3xy^2, xe^z, z^3\rangle$. Calculate the flux of $\vec{F}$ across the surface S, assuming it has positive orientation.

3. Let S be the surface $x^2 + y^2 + z^2 = 4$ with positive orientation and let $\vec{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3\rangle$. Evaluate the surface integral
   $$\iint_S \vec{F} \cdot d\vec{S}.$$

4. Compute $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$ if $\vec{F} = \langle x^2y, -xy^2, 4z(z^2 - 1)\rangle$ if S is the unit cube with outward orientation formed by the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$,
   $(1, 1, 0)$, $(0, 0, 1)$, $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$, minus both the top and bottom faces.
   Hint: Think about the behavior of the vector field $\vec{F}$ on the top and bottom faces. Where would a normal vector point there?

5. Let $\vec{F}(x, y, z) = \langle P, Q, R\rangle$ be a vector field and S a closed surface with positive orientation containing a region E. Use the Divergence Theorem to compute
   $$\iint_S \text{curl}\vec{F} \cdot d\vec{S}.$$