Math 1071 Spring 2016, Final Exam Review Questions

This review packet only covers the sections from Chapter 6 covered after Exam 2. ***The final exam will be cumulative.*** *You should look at the review packet for Exams 1 and 2, as well as past assignments, quizzes, worksheets, and relevant sections of our text to remind yourself of earlier material. This review packet is not intended to be exhaustive.*

1. Evaluate the following indefinite integrals.

   (a) $\int 5\,dx$

   (b) $\int x^{99}\,dx$

   (c) $\int x^{-99}\,dx$

   (d) $\int \frac{5}{x^3}\,dy$

   (e) $\int \frac{y^{\frac{3}{2}}}{\sqrt{2}}\,dy$

   (f) $\int \sqrt[3]{u^2}\,du$

   (g) $\int (6x^2 + 4x)\,dx$

   (h) $\int \left(\frac{3}{t^2} - 6t^2\right)\,dt$

   (i) $\int \left(x + \frac{1}{x^3}\right)\,dx$

   (j) $\int \left(\pi + \frac{1}{x}\right)\,dx$

   (k) $\int \frac{t+1}{\sqrt{t}}\,dt$

   (l) $\int \left(e^x - \frac{1}{x^2}\right)\,dx$

   (m) $\int (u^2+1)(3-u)\,du$

   (n) $\int t(\sqrt{t} - t^4)\,dt$

2. Evaluate the following indefinite integrals.

   (a) $\int 6(3x+1)^{10}\,dx$
   (b) $\int x(3-x^2)^6\,dx$
   (c) $\int (x^3+2)\sqrt[3]{x^4+8x+3}\,dx$
   (d) $\int 2\sqrt{x+1}\,dx$
   (e) $\int \sqrt[3]{(x+1)^2}\,dx$
   (f) $\int \frac{\ln 2x}{x}\,dx$
   (g) $\int \frac{1}{2x+1}\,dx$
   (h) $\int \frac{e^{-x}}{e^{-x}+1}\,dx$
   (i) $\int \frac{1}{x\ln x}\,dx$
   (j) $\int \frac{1}{x\ln x^2}\,dx$

3. Use a left- and right-hand sum with rectangles of equal width for the given value of n to approximate the integral. Round the answers to two decimal places.

   (a) $\int_1^2 \ln x\,dx$, $n=3$
   (b) $\int_0^2 xe^x\,dx$, $n=4$
   (c) $\int_{-1}^1 \sqrt[3]{3-x^2}\,dx$, $n=4$

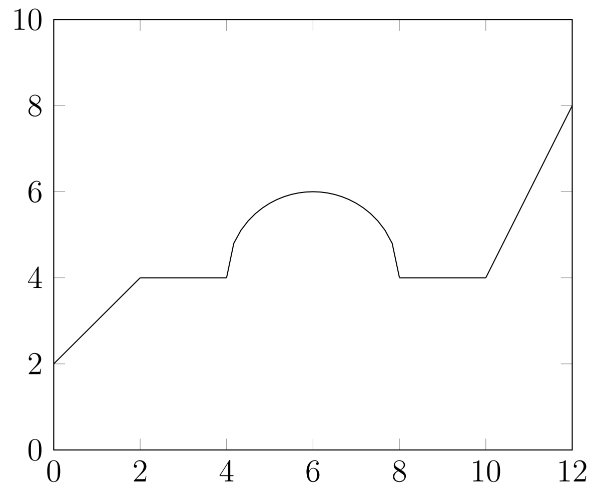
4. Evaluate the following definite integrals.

   (a) $\int_1^2 4x^3\,dx$
   (b) $\int_{-2}^{-2} 3x^4\,dx$
   (c) $\int_{-1}^0 (9x^2-1)\,dx$
   (d) $\int_1^2 (x^{-2}+3x^{-4})\,dx$
   (e) $\int_{-1}^0 e^{-x}\,dx$
   (f) $\int_{-2}^{-1} e^{2x}\,dx$
   (g) $\int_2^4 \frac{3}{x}\,dx$
   (h) $\int_{-1}^0 (1+2x)^5\,dx$
   (i) $\int_{-1}^1 xe^{x^2+1}\,dx$
   (j) $\int_1^2 \frac{1}{2x+1}\,dx$

(k) $\int_{-2}^{0} \frac{x}{x^2+1}\, dx$

(l) $\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

5. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



(a) $\int_0^4 f(x)\, dx$

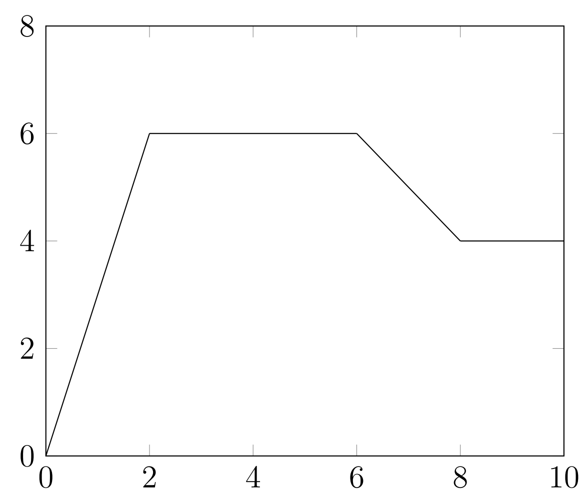
(b) $\int_2^8 f(x)\, dx$

(c) $\int_{10}^{12} f(x)\, dx$

6. (a) If $f(6) = 15$, f' is continuous, and $\int_6^{10} f'(x)\, dx = 23$, what is the value of $f(10)$?

   (b) If $f(4) = 20$, f' is continuous, and $\int_1^4 f'(x)\, dx = 6$, what is the value of $f(1)$?

7. Using the graph of $f'(x)$, given below, and the fact that $f(0) = 12$, compute the following values.



(a) $f(2)$

(b) $f(5)$

(c) $f(9)$

8. Suppose the rate of sales of an item is given by

$$S'(t) = -3t^2 + 36t$$

where t is the number of weeks after an advertising campaign has begun. How many items were sold during the third week?

9. A tank holding 10,000 gallons of a polluting chemical breaks at the bottom and spills out at the rate given by

$$f'(t) = 400e^{-0.01t},$$

where t is measured in hours. How much spills during the first day?

10. Find the average value of the given function on the given interval.

    (a) $f(x) = 2x$ on $[-2, 2]$

    (b) $f(x) = x^3$ on $[0, 3]$

    (c) $f(x) = e^x$ on $[0, \ln 2]$

    (d) $f(x) = \frac{4}{x}$ on $[1, e^2]$

    (e) $f(x) = x(x - 1)$ on $[0, 2]$

11. The population in a certain city is projected to be given by

$$P(t) = 100,000e^{0.05t},$$

where t is given in years from now. Find the average population over the next 10 years.

12. The profit in millions of dollars of a certain firm is given by

$$P(t) = 6(t - 1)(t - 2),$$

where t is measured in years. Find the average profit per year over the period of time $[0, 4]$.

13. Find the area of the region enclosed by the given curves.

(a) $y = x^3, y = 0, x = 1, x = 2$

(b) $y = e^{-x}, y = 3, x = -1, x = 0$

(c) $y = \sqrt{x}, y = x, x = 0, x = 1$

(d) $y = x^2 - 2x + 1, y = x + 1$

(e) $y = x^2, y = 8 - x^2$

(f) $y = e^{2x}, y = e^{-2x}, x = -1, x = 2$

(g) $y = x^3 - 3x, y = 2x^2$

(h) $y = x^3 - 3x, y = -2x^3$

(i) $y = \sqrt[3]{x}, y = x, x = -8, x = 1$