Math 1071 Spring 2016, Final Exam Review Solutions

This review packet only covers the sections from Chapter 6 covered after Exam 2. ***The final exam will be cumulative.*** *You should look at the review packet for Exams 1 and 2, as well as past assignments, quizzes, worksheets, and relevant sections of our text to remind yourself of earlier material. This review packet is not intended to be exhaustive.*

1. For each of the answers below, we use C as the constant of integration.

   (a) $\int 5\,dx = 5x + C$

   (b) $\int x^{99}\,dx = \frac{1}{100}x^{100} + C$

   (c) $\int x^{-99}\,dx = -\frac{1}{98}x^{-98} + C$

   (d) $\int \frac{5}{x^3}\,dy = 5\int x^{-3}dx = 5(-\frac{1}{2}x^{-2}) + C = -\frac{5}{2}x^{-2} + C$

   (e) $\int \frac{y^{\frac{3}{2}}}{\sqrt{2}}\,dy = \frac{1}{\sqrt{2}}\int y^{\frac{3}{2}}dy = \frac{1}{\sqrt{2}}\left[\left(\frac{1}{(5/2)}\right)y^{5/2}\right] + C = \frac{1}{\sqrt{2}}(\frac{2}{5})y^{\frac{5}{2}} + C = \frac{2}{5\sqrt{2}}y^{\frac{5}{2}} + C$

   (f) $\int \sqrt[3]{u^2}\,du = \int u^{\frac{2}{3}}du = \frac{1}{(5/3)}u^{\frac{5}{3}} + C = \frac{3}{5}u^{\frac{5}{3}} + C$

   (g) $\int(6x^2 + 4x)\,dx = 6\int x^2dx + 4\int xdx = 2x^3 + 2x^2 + C$

   (h) $\int\left(\frac{3}{t^2} - 6t^2\right)\,dt = 3\int t^{-2}dt - 6\int t^2dt = 3(-t^{-1}) - 2t^3 + C = -3t^{-1} - 2t^3 + C$

   (i) $\int\left(x + \frac{1}{x^3}\right)\,dx = \int x\,dx + \int x^{-3}dx = \frac{1}{2}x^2 + -\frac{1}{2}x^{-2} + C$

   (j) $\int\left(\pi + \frac{1}{x}\right)\,dx = \int \pi\,dx + \int \frac{1}{x}\,dx = \pi x + \ln|x| + C$

   (k) $\int \frac{t+1}{\sqrt{t}}\,dt = \int\left(\frac{t}{\sqrt{t}} + \frac{1}{\sqrt{t}}\right)\,dt = \int(t^{\frac{1}{2}} + t^{-\frac{1}{2}})\,dt = \int t^{\frac{1}{2}}\,dt + \int t^{-\frac{1}{2}}\,dt$
   $= \frac{1}{(3/2)}t^{\frac{3}{2}} + \frac{1}{(1/2)}t^{\frac{1}{2}} + C = \frac{2}{3}t^{\frac{3}{2}} + 2t^{\frac{1}{2}} + C$

   (l) $\int\left(e^x - \frac{1}{x^2}\right)\,dx = \int e^x\,dx - \int x^{-2}\,dx = e^x + x^{-1} + C$

   (m) $\int(u^2+1)(3-u)\,du = \int(3u^2 - u^3 + 3 - u)\,du = 3\int u^2\,du - \int u^3\,du + 3\int du - \int u\,du$
   $= u^3 - \frac{1}{4}u^4 + 3u - \frac{1}{2}u^2 + C$

   (n) $\int t(\sqrt{t} - t^4)\,dt = \int(t^{\frac{3}{2}} - t^5)\,dt = \int t^{\frac{3}{2}}\,dt - \int t^5\,dt = \frac{1}{5/2}t^{\frac{5}{2}} - \frac{1}{6}t^6 + C = \frac{2}{5}t^{\frac{5}{2}} - \frac{1}{6}t^6 + C$

2. For each of the answers below, we use C as the constant of integration.

   (a) Let $u = 3x + 1$. Then $du = 3dx$, so $dx = \frac{1}{3}du$.
   Then $\int 6(3x + 1)^{10}\,dx = \int 2u^{10}\,du = \frac{2}{11}u^{11} + C = \frac{2}{11}(3x + 1)^{11} + C$.

   (b) Let $u = 3 - x^2$. Then $du = -2xdx$ and $-\frac{1}{2}du = x\,dx$.
   Then $\int x(3-x^2)^6\,dx = -\frac{1}{2}\int u^7\,du = -\frac{1}{2}(\frac{1}{7}u^7) + C = -\frac{1}{14}u^7 + C = -\frac{1}{14}(3-x^2)^7 + C$

   (c) Let $u = x^4+8x+3$. Then $du = (4x^3+8)\,dx = 4(x^3+2)\,dx$. Then $\frac{1}{4}\,du = (x^3+2)\,dx$,
   so $\int (x^3 + 2)\sqrt[3]{x^4 + 8x + 3}\,dx = \frac{1}{4}\int \sqrt[3]{u}\,du = \frac{1}{4}\int u^{\frac{1}{3}}\,du = \frac{1}{4}\left(\frac{1}{4/3}u^{\frac{4}{3}}\right) + C$
   $= \frac{3}{16}(x^4 + 8x + 3) + C$

   (d) Let $u = x + 1$. Then $du = dx$, so
   $\int 2\sqrt{x+1}\,dx = 2\int \sqrt{u}\,du = 2\int u^{\frac{1}{2}}\,du = 2\left(\frac{1}{3/2}u^{\frac{3}{2}}\right) + C = \frac{4}{3}(x+1)^{\frac{3}{2}} + C$

   (e) Let $u = x + 1$. Then $du = dx$, so
   $\int \sqrt[3]{(x+1)^2}\,dx = \int (x+1)^{\frac{2}{3}}\,dx = \int u^{\frac{2}{3}}\,du = \frac{1}{(5/3)}u^{\frac{5}{3}}\,du + C = \frac{3}{5}(x+1)^{\frac{5}{3}} + C$

   (f) Let $x = \ln 2x$. Then $du = \frac{1}{x}\,dx$ so
   $\int \frac{\ln 2x}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln 2x)^2 + C$

   (g) Let $x = 2x + 1$. The $du = 2\,dx$ and $\frac{1}{2}\,du = dx$. So
   $\int \frac{1}{2x+1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x + 1| + C$

   (h) Let $u = e^{-x} + 1$ and $du = -e^{-x}\,dx$. Then
   $\int \frac{e^{-x}}{e^{-x}+1}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|e^{-x} + 1| + C = -\ln(e^{-x} + 1) + C$
   (since $e^{-x} + 1 > 0$ for all x)

   (i) Let $u = \ln x$. Then $du = \frac{dx}{x}$ and
   $\int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$

   (j) Let $u = \ln x^2$. Then $du = \frac{2}{x}\,dx$ and
   $\int \frac{1}{x\ln x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|\ln x^2| + C$

3. Use a left- and right-hand sum with rectangles of equal width for the given value of n to approximate the integral. Round the answers to two decimal places.

   (a) $\int_1^2 \ln x\,dx$, $n = 3$
   Let $f(x) = \ln x$. When $n = 3$, $\Delta x = \frac{2-1}{3} = \frac{1}{3}$.
   Left-hand sum:

$$\begin{aligned}
\int_1^2 \ln x\, dx &\approx \frac{1}{3}\cdot f(1) + \frac{1}{3}\cdot f\left(\frac{4}{3}\right) + \frac{1}{3}\cdot f\left(\frac{5}{3}\right) \\
&= \frac{1}{3}\cdot \ln(1) + \frac{1}{3}\cdot \ln\left(\frac{4}{3}\right) + \frac{1}{3}\cdot \ln\left(\frac{5}{3}\right) \\
&\approx 0.266169
\end{aligned}$$

Right-hand sum:

$$\begin{aligned}
\int_1^2 \ln x\, dx &\approx \frac{1}{3}\cdot f\left(\frac{4}{3}\right) + \frac{1}{3}\cdot f\left(\frac{5}{3}\right) + \frac{1}{3}f(2) \\
&= \frac{1}{3}\cdot \ln\left(\frac{4}{3}\right) + \frac{1}{3}\cdot \ln\left(\frac{5}{3}\right) + \frac{1}{3}\ln(2) \\
&\approx 0.497218
\end{aligned}$$

(b) $\int_0^2 xe^x\, dx$, $n = 4$

Let $f(x) = xe^x\, dx$. When $n = 4$, $\Delta x = \frac{2-0}{4} = \frac{1}{2}$.

Left-hand sum:

$$\begin{aligned}
\int_0^2 xe^x\, dx &\approx \frac{1}{2}\cdot f(0) + \frac{1}{2}\cdot f\left(\frac{1}{2}\right) + \frac{1}{2}\cdot f(1) + \frac{1}{2}\cdot f\left(\frac{3}{2}\right) \\
&= \frac{1}{2}\cdot (0)e^{(0)} + \frac{1}{2}\cdot \frac{1}{2}e^{(1/2)} + \frac{1}{2}\cdot (1)e^{(1)} + \frac{1}{2}\cdot (\frac{3}{2})e^{(3/2)} \\
&\approx 5.13259
\end{aligned}$$

Right-hand sum:

$$\begin{aligned}
\int_0^2 xe^x\, dx \frac{1}{2}\cdot f\left(\frac{1}{2}\right) + \frac{1}{2}\cdot f(1) + \frac{1}{2}\cdot f\left(\frac{3}{2}\right) &\approx \frac{1}{2}\cdot f(2) \\
&= \frac{1}{2}\cdot \frac{1}{2}e^{(1/2)} + \frac{1}{2}\cdot (1)e^{(1)} + \frac{1}{2}\cdot (\frac{3}{2})e^{(3/2)} + \frac{1}{2}\cdot (2)e^{(2)} \\
&\approx 12.5216
\end{aligned}$$

(c) $\int_{-1}^1 \sqrt[3]{3-x^2}\, dx$, $n = 4$

Let $f(x) = \sqrt[3]{3-x^2}$. When $n = 4$, $\Delta x = \frac{1-(-1)}{4}$.

Left-hand sum:

$$\begin{aligned}\int_{-1}^{1}\sqrt[3]{3-x^2}\,dx &\approx \frac{1}{2}\cdot f(-1)+\frac{1}{2}\cdot f\left(-\frac{1}{2}\right)+\frac{1}{2}\cdot f(0)+\frac{1}{2}\cdot f\left(\frac{1}{2}\right)\\ &= \frac{1}{2}\sqrt[3]{3-(-1)^2}+\frac{1}{2}\cdot\sqrt[3]{3-(-1/2)^2}+\frac{1}{2}\cdot\sqrt[3]{3-(0)^2}+\frac{1}{2}\cdot\sqrt[3]{3-(1/2)^2}\\ &\approx 2.7521\end{aligned}$$

Right-hand sum:

$$\begin{aligned}\int_{-1}^{1}\sqrt[3]{3-x^2}\,dx &\approx \frac{1}{2}\cdot f\left(-\frac{1}{2}\right)+\frac{1}{2}\cdot f(0)+\frac{1}{2}\cdot f\left(\frac{1}{2}\right)+\frac{1}{2}\cdot f(1)\\ &= \frac{1}{2}\cdot\sqrt[3]{3-(-1/2)^2}+\frac{1}{2}\cdot\sqrt[3]{3-(0)^2}+\frac{1}{2}\cdot\sqrt[3]{3-(1/2)^2}+\frac{1}{2}\sqrt[3]{3-(1)^2}\\ &\approx 2.7521\end{aligned}$$

4. Evaluate the following definite integrals.

   (a) $\int_1^2 4x^3\,dx = x^4\big|_1^2 = 2^4-1^4 = 15$

   (b) $\int_{-2}^{-2} 3x^4\,dx = 0$

   (c) $\int_{-1}^{0}(9x^2-1)\,dx = 3x^3-x\big|_{-1}^{0} = (3(0)^3-0)-(3(-1)^3-(-1)) = 2$

   (d) $\int_1^2(x^{-2}+3x^{-4})\,dx = \left(-\frac{1}{x}-\frac{1}{x^3}\right)\Big|_1^2 = \left(-\frac{1}{2}-\frac{1}{2^3}\right)-\left(-\frac{1}{1}-\frac{1}{1^3}\right) = \frac{11}{8}$

   (e) $\int_{-1}^{0} e^{-x}\,dx = (-e^{-x})\big|_{-1}^{0} = -e^0+e^1 = -1+e$

   (f) $\int_{-2}^{-1} e^{2x}\,dx = \left(\frac{1}{2}e^{2x}\right)\Big|_{-2}^{-1} = \frac{1}{2}e^{-2}-\frac{1}{2}e^{-4}$

   (g) $\int_2^4 \frac{3}{x}\,dx = (3\ln|x|)\big|_2^4 = 3\ln 4-3\ln 2$

   (h) $\int_{-1}^{0}(1+2x)^5\,dx = \frac{1}{2}\left((1+2x)^6\right)\big|_{-1}^{0} = \frac{1}{2}(1+0)^6-\frac{1}{2}(1-2)^6 = \frac{1}{2}-\frac{1}{2} = 0$

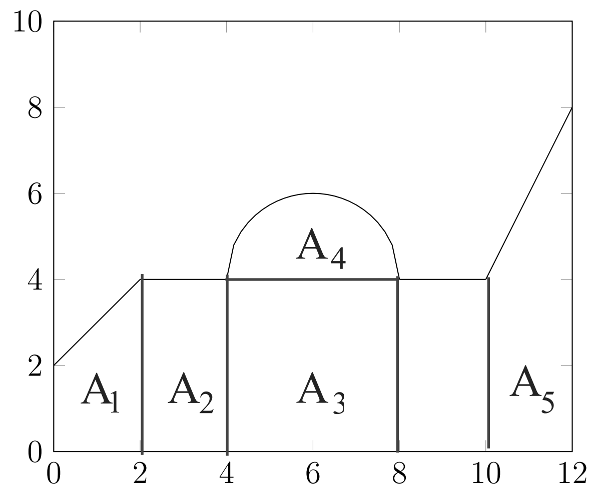
   (i) $\int_{-1}^{1} xe^{x^2+1}\,dx = \left(\frac{1}{2}e^{x^2+1}\right)\Big|_{-1}^{1} = \frac{1}{2}e^2-\frac{1}{2}e^2 = 0$

   (j) $\int_1^2 \frac{1}{2x+1}\,dx = \left(\frac{1}{2}\ln|2x+1|\right)\Big|_1^2 = \frac{1}{2}\ln 5-\frac{1}{2}\ln 3$

   (k) $\int_{-2}^{0} \frac{x}{x^2+1}\,dx = \left(\frac{1}{2}\ln(x^2+1)\right)\Big|_{-2}^{0} = \frac{1}{2}\ln 1-\frac{1}{2}\ln 5 = -\frac{1}{2}\ln 5$

   (l) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \left(2e^{\sqrt{x}}\right)\Big|_1^4 = 2e^2-2e$

5. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



For the following solutions, the areas below the graph have been labeled.

You may find the following formulas helpful:

- Area of a triangle $= \frac{1}{2}$(base)(height)
- Area of a rectangle = (base)(height)
- Area of a circle $= \pi(\text{radius})^2$
- Area of a trapezoid $= \frac{1}{2}(\text{height}_1 + \text{height}_2)(\text{base})$

(a) $\int_0^4 f(x)\,dx = A_1 + A_2 = \frac{1}{2}(2+4)(2) + (2)(4) = 14$

(b) $\int_2^8 f(x)\,dx = A_2 + A_3 + A_4 = (2)(4) + (4)(4) + \frac{1}{2}\pi(2)^2 = 24 + 2\pi$

(c) $\int_{10}^{12} f(x)\,dx = A_5 = \frac{1}{2}2(4+8) = 12$

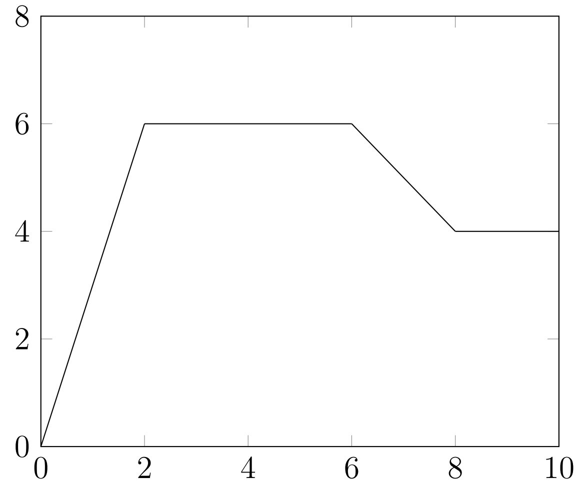
6. (a) If $f(6) = 15$, f' is continuous, and $\int_6^{10} f'(x)\,dx = 23$, what is the value of $f(10)$?

   By the Fundamental Theorem of Calculus, part 2, $\int_6^{10} f'(x)\,dx = f(10) - f(6)$ so $23 = f(10) - 15$. Then $f(10) = 38$.

   (b) If $f(4) = 20$, f' is continuous, and $\int_1^4 f'(x)\,dx = 6$, what is the value of $f(1)$?

   By the Fundamental Theorem of Calculus, part 2, $\int_1^4 f'(x)\,dx = f(4) - f(1)$, so $6 = 20 - f(1)$. Then $f(1) = 14$.

7. Using the graph of $f'(x)$, given below, and the fact that $f(0) = 12$, compute the following values.



(a) We can see graphically that $\int_0^2 f'(x)\,dx = \frac{1}{2}(2)(6) = 6$. By the Fundamental Theorem of Calculus, part 2, we also know $\int_0^2 f'(x)\,dx = f(2) - f(0)$. Then $6 = f(2) - 12$. Thus $f(2) = 18$

(b) We can see graphically that $\int_0^5 f'(x)\,dx = \frac{1}{2}(2)(6) + (3)(6) = 24$. By the Fundamental Theorem of Calculus, part 2, we also know $\int_0^5 f'(x)\,dx = f(5) - f(0)$. Then $24 = f(5) - 12$. Thus $f(5) = 36$

(c) We can see graphically that $\int_0^9 f'(x)\,dx = \frac{1}{2}(2)(6) + (4)(6) + \frac{1}{2}(2)(6+4) + (1)(4) = 44$. By the Fundamental Theorem of Calculus, part 2, we also know $\int_0^9 f'(x)\,dx = f(9) - f(0)$. Then $44 = f(9) - 12$, so $f(9) = 56$.

8. Suppose the rate of sales of an item is given by

$$S'(t) = -3t^2 + 36t$$

where t is the number of weeks after an advertising campaign has begun. How many items were sold during the third week?

The number of items sold during the third week is given by

$$\begin{aligned}
\int_2^3 S'(t)\,dt &= \int_2^3 (-3t^2 + 36t)\,dt \\
&= \left(-t^3 + 18t^2\right)\Big|_2^3 \\
&= \left(-(3)^3 + 18(3)^2\right) - \left(-(2)^3 + 18(2)^2\right) \\
&= 71
\end{aligned}$$

9. A tank holding 10,000 gallons of a polluting chemical breaks at the bottom and spills out at the rate given by

$$f'(t) = 400e^{-0.01t},$$

where t is measured in hours. How much spills during the first day?

The amount of oil lost spilled during the first day is

$$\begin{aligned}\int_0^{24} f'(t)\,dt &= \int_0^{24} 400e^{-0.01t}\,dt \\ &= \left(-40000e^{0.01t}\right)\Big|_0^{24} \\ &= \left(-40000e^{0.01(24)}\right) - \left(-40000e^{0.01(0)}\right) \\ &\approx 8534.89 \text{ gallons}\end{aligned}$$

10. For the following solutions, we denote the average value of $f(x)$ by $\overline{f}(x)$

(a) $\overline{f}(x) = \frac{1}{2-(-2)} \int_{-2}^{2} 2x\,dx = \frac{1}{4}\,x^2\big|_{-2}^{2} = \frac{1}{4}2^2 - (-2)^2 = 0$

(b) $\overline{f}(x) = \frac{1}{3-0} \int_0^3 x^3\,dx = \frac{1}{3} \cdot \frac{1}{4}x^4\big|_0^3 = \frac{1}{3}\left[\frac{1}{4}(3)^4 - \frac{1}{4}(0)^4\right] = \frac{81}{12}$

(c) $\overline{f}(x) = \frac{1}{\ln 2 - 0} \int_0^{\ln 2} e^x\,dx = \frac{1}{\ln 2}\,e^x\big|_0^{\ln 2} = \frac{1}{\ln 2}\left[e^{\ln 2} - e^0\right] = \frac{1}{\ln 2}(2-1) = \frac{1}{\ln 2}$

(d) $\overline{f}(x) = \frac{1}{e^2-1} \int_1^{e^2} \frac{4}{x}\,dx = \frac{1}{e^2-1} \cdot 4\ln|x|\big|_1^{e^2} = \frac{1}{e^2-1}\left[4\ln|e^2| - 4\ln|1|\right] = \frac{1}{e^2-1}(8-0) = \frac{8}{e^2-1}$

(e) $\overline{f}(x) = \frac{1}{2-0} \int_0^2 x(x-1)\,dx = \frac{1}{2}\int_0^2 (x^2 - x)\,dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_0^2$

$= \frac{1}{2}\left[\left(\frac{(2)^3}{3} - \frac{(2)^2}{2}\right) - \left(\frac{(0)^3}{3} - \frac{(0)^2}{2}\right)\right] = \frac{1}{3}$

11. The population in a certain city is projected to be given by

$$P(t) = 100000e^{0.05t},$$

where t is given in years from now. Find the average population over the next 10 years.

The average population over the next 10 years is

$$\begin{aligned}\frac{1}{10-0}\int_0^{10} P(t)\,dt &= \frac{1}{10}\int_0^{10} 100,000e^{0.05t}\,dt \\ &= \frac{1}{10}\left(2000000e^{t/20}\right)\Big|_1^{10} \\ &\approx 129744\end{aligned}$$

12. The profit in millions of dollars of a certain firm is given by

$$P(t) = 6(t-1)(t-2),$$

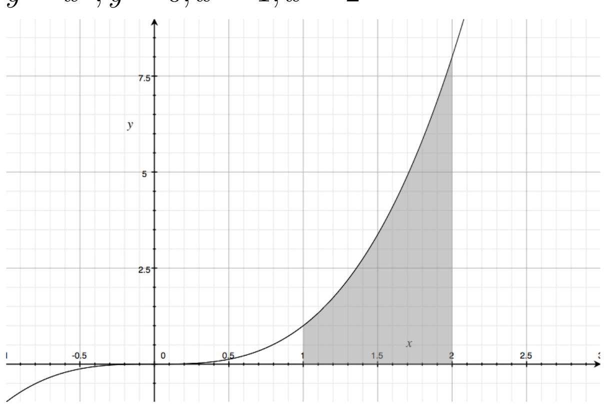
where t is measured in years. Find the average profit per year over the period of time $[0, 4]$.

The average profit in millions of dollars is given by

$$\begin{aligned}
\frac{1}{4-0}\int_0^4 P(t)\,dt &= \frac{1}{4}\int_0^4 6(t-1)(t-2)\,dt \\
&= \frac{3}{2}\int_0^4 (t^2-3t+2)\,dt \\
&= \frac{3}{2}\left(\frac{t^3}{3}-\frac{3t^2}{2}+2t\right)\Bigg|_0^4 \\
&= \frac{3}{2}\left[\left(\frac{4^3}{3}-\frac{3(4)^2}{2}+2(4)\right)-\left(\frac{(0)^3}{3}-\frac{3(0)^2}{2}+2(0)\right)\right] \\
&= \frac{3}{2}\left(\frac{16}{3}\right) \\
&= 8
\end{aligned}$$

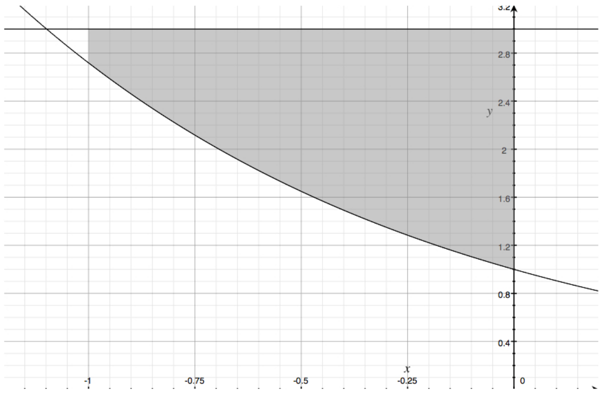
13. Find the area of the region enclosed by the given curves.

(a) $y = x^3, y = 0, x = 1, x = 2$



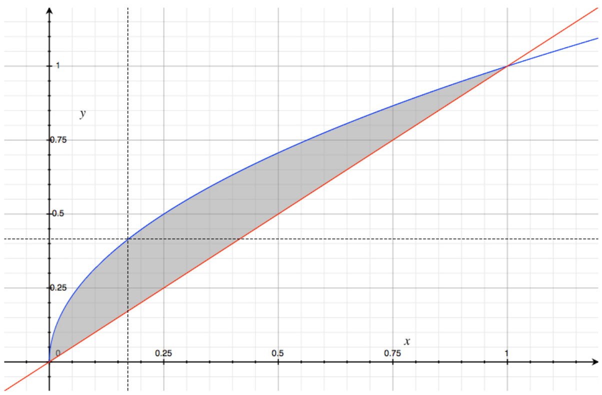
$A = \int_1^2 x^3\,dx = \frac{x^4}{4}\Big|_1^2 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$

(b) $y = e^{-x}, y = 3, x = -1, x = 0$



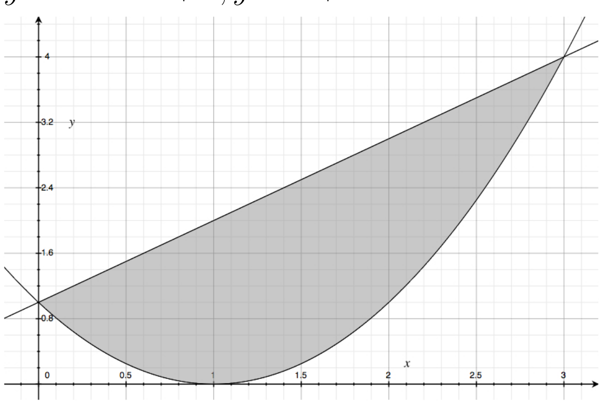
$A = \int_{-1}^{0}(3 - e^{-x})\,dx = (3x + e^{-x})\big|_{-1}^{0} = \left(3(0) + e^{-(0)}\right) - \left(3(-1) + e^{-(-1)}\right) = 4 - e$

(c) $y = \sqrt{x}, y = x, x = 0, x = 1$



$A = \int_{0}^{1}(\sqrt{x} - x)\,dx = \left(\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right)\Big|_{0}^{1} = \frac{1}{6}$

(d) $y = x^2 - 2x + 1, y = x + 1$



To find the bounds for integration, we determine where the two given curves intersect:

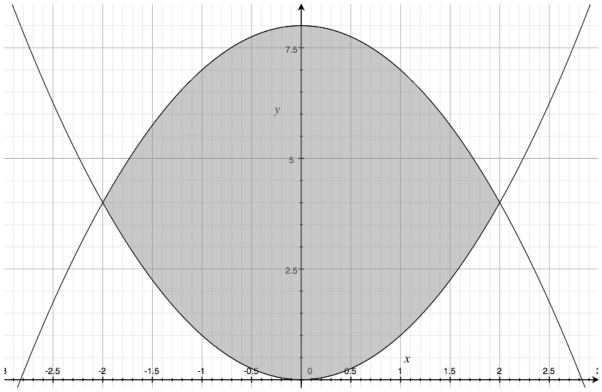
$x^2 - 2x + 1 = x + 1 \Rightarrow x^2 - 3x = 0 \Rightarrow x(x - 3) = 0 \Rightarrow x = 0, x = 3$

Then

$A = \int_{0}^{3}[(x + 1) - (x^2 - 2x + 1)]\,dx = \int_{0}^{3}(-x^2 + 3x)\,dx = \left(\frac{-x^3}{3} + \frac{3x^2}{2}\right)\Big|_{0}^{3}$

$= \left(\frac{-(3)^3}{3} + \frac{3(3)^2}{2}\right) - \left(\frac{-(0)^3}{3} + \frac{3(0)^2}{2}\right) = \frac{9}{2}$

(e) $y = x^2, y = 8 - x^2$



To find the bounds for integration, determine where the two given curves intersect:

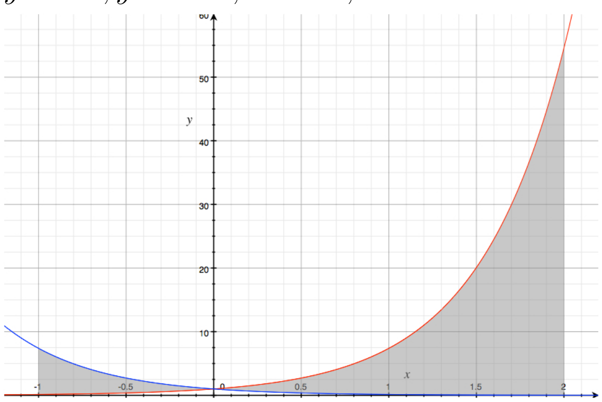
$x^2 = 8 - x^2 \Rightarrow 2x^2 - 8 = 0 \Rightarrow 2(x^2 - 4) = 0 \Rightarrow x = -2, x = 2$

Then

$A = \int_{-2}^{2} [(8 - x^2) - x^2]\, dx = \int_{-2}^{2} (8 - 2x^2)\, dx = \left(8x - \frac{2x^3}{3}\right)\Big|_{-2}^{2}$

$= \left(8(2) - \frac{2(2)^3}{3}\right) - \left(8(-2) - \frac{2(-2)^3}{3}\right) = \frac{64}{3}$

(f) $y = e^{2x}, y = e^{-2x}, x = -1, x = 2$



Observe that the roles of "top function" and "bottom function" switch when the two functions intersect, so in order to find the area between the curves, we must find the two areas separately and then add them together.

The graphs intersect when $e^{2x} = e^{-2x} \Rightarrow 2x = -2x \Rightarrow 4x = 0 \Rightarrow x = 0$.

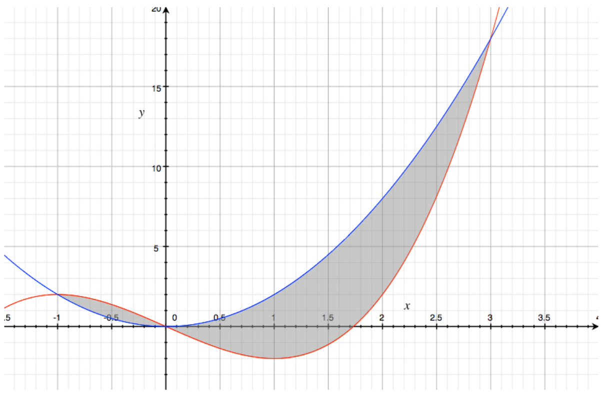
Then $A = \int_{-1}^{0} (e^{-2x} - e^{2x})\, dx + \int_{0}^{2} (e^{2x} - e^{-2x})\, dx$

$= \left(-\frac{1}{2}e^{-2x} - \frac{1}{2}e^{2x}\right)\Big|_{-1}^{0} + \left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)\Big|_{0}^{2}$

$= \left[\left(-\frac{1}{2} - \frac{1}{2}\right) - \left(-\frac{1}{2}e^{-2(-1)} - \frac{1}{2}e^{2(1)}\right)\right] + \left[\left(\frac{1}{2}e^{2(2)} + \frac{1}{2}e^{-2(2)}\right) - \left(\frac{1}{2} + \frac{1}{2}\right)\right]$

$= \frac{1}{2}e^2 + \frac{1}{2}e^{-2} + \frac{1}{2}e^4 + \frac{1}{2}e^{-4} - 2$

(g) $y = x^3 - 3x, y = 2x^2$



As in the example above, the roles of the "top function" and "bottom function" switch when the two functions intersect, so in order to find the area between the curves, we must find two areas separately and then add them together. We also need to find the intersection in order to find the bounds for our integrations.

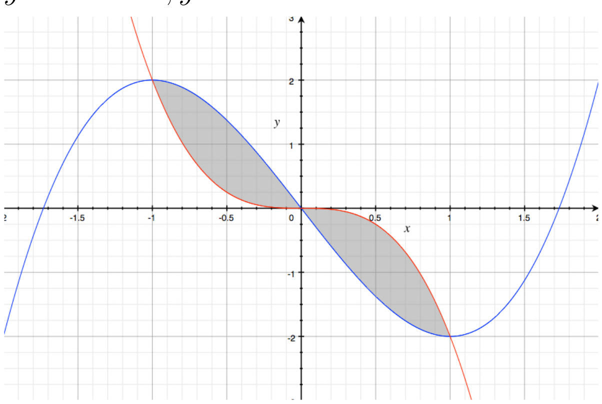
The graphs intersect when

$x^3 - 3x = 2x^2 \Rightarrow x^3 - 2x^2 - 3x = 0 \Rightarrow x(x+1)(x-3) = 0 \Rightarrow x = 0, x = -1, x = 3.$

Then $A = \int_{-1}^{0} [(x^3 - 3x) - 2x^2]\, dx + \int_0^3 [2x^2 - (x^3 - 3x)]\, dx$

$$= \left(\frac{x^4}{4} - \frac{3x^2}{2} - \frac{2x^3}{3}\right)\Big|_{-1}^{0} + \left(\frac{2x^3}{3} - \frac{x^4}{4} + \frac{3x^2}{2}\right)\Big|_{0}^{3} = \frac{7}{12} + \frac{45}{4} = \frac{71}{6}$$

(h) $y = x^3 - 3x, y = -2x^3$



Again, the roles of the "top function" and "bottom function" switch when the two functions intersect, so in order to find the area between the curves, we must find two areas separately and then add them together. We also need to find the intersection in order to find the bounds for our integrations.

The graphs intersect when

$x^3 - 3x = -2x^3 \Rightarrow 3x^3 - 3x = 0 \Rightarrow 3x(x^2 - 1) = 0 \Rightarrow x = -1, x = 0, x = 1$

Then $A = \int_{-1}^{0} [(x^3 - 3x) - (-2x^3)]\, dx + \int_{0}^{1} [-2x^3 - (x^3 - 3x))\, dx$
$= \int_{-1}^{0} [3x^3 - 3x]\, dx + \int_{0}^{1} [-3x^3 + 3x]\, dx$
$= \left(\frac{3x^4}{4} - \frac{3x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{-3x^4}{4} + \frac{3x^2}{2}\right)\Big|_{0}^{1}$
$= \left(0 - \left(\frac{3}{4} - \frac{3}{2}\right)\right) + \left(\left(\frac{-3}{4} + \frac{3}{2}\right) - 0\right) = \frac{3}{2}$

(i) $y = \sqrt[3]{x}, y = x, x = -8, x = 1$

The bounds for x are given, but we still need to note that the "top function" and "bottom function" switch roles when the functions intersect. In this case they intersect at 3 points:

The graphs intersect when $\sqrt[3]{x} = x \Rightarrow x = x^3 \Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$
$\Rightarrow x = 0, x = 1, x = -1.$

Then $A = \int_{-8}^{-1} (\sqrt[3]{x} - x)\, dx + \int_{-1}^{0} (x - \sqrt[3]{x})\, dx + \int_{0}^{1} (\sqrt[3]{x} - x)\, dx$
$= \left(\frac{3}{4}x^{\frac{4}{3}} - \frac{x^2}{2}\right)\Big|_{-8}^{-1} + \left(\frac{x^2}{2} - \frac{3}{4}x^{\frac{4}{3}}\right)\Big|_{-1}^{0} + \left(\frac{3}{4}x^{\frac{4}{3}} - \frac{x^2}{2}\right)\Big|_{0}^{1} = \frac{81}{4} + \frac{1}{4} + \frac{1}{4} = \frac{83}{4}$