Math 1071 Spring 2016, Exam 2 Review Answers

1. For parts (a)-(c) below, we use tangent line approximation $f(x) \approx f(c) + f'(c)(x - c)$.

   (a) Using $f(x) = \sqrt{x}$, $c = 16$ and $x = 15.7$, we see $f'(x) = \frac{1}{2\sqrt{x}}$, so

$$\begin{aligned}
\sqrt{15.7} &= f(15.7) \\
&\approx f(16) + f'(16)(15.7 - 16) \\
&= \sqrt{16} + \frac{1}{2\sqrt{16}}(-0.3) \\
&= 4 + \frac{1}{8}(-0.3) \\
&= 3.9625
\end{aligned}$$

   (b) Using $f(x) = x^3$, $c = 2$ and $x = 1.8$, we see $f'(x) = 3x^2$, so

$$\begin{aligned}
(1.8)^3 &= f(1.8) \\
&\approx f(2) + f'(2)(1.8 - 2) \\
&= 2^3 + 3(2)^2(-0.2) \\
&= 8 + 3 \cdot 4(-0.2) \\
&= 5.6
\end{aligned}$$

(c) Using $f(x) = \ln x$, $c = 1$, and $x = 1.2$, we see $f'(x) = 1/x$, so

$$\begin{aligned} \ln 1.2 &= f(1.2) \\ &\approx f(1) + f'(1.2)(1.2 - 1) \\ &= \ln 1 + \frac{1}{1}(0.2) \\ &= 0 + 0.2 \\ &= 0.2 \end{aligned}$$

2. The linear approximation formula is given by $f(c + h) - f(c) \approx f'(c)h$. Since we are interested in the change of area of a square when side length increases from 3 inches to 3.2 inches, we use the function $f(x) = x^2$, where x denotes the length of a side of a square, $c = 3$ and $h = 3.2 - 3 = 0.2$. Then the change in area is given by

$$f(3.2) - f(3) \approx f'(3)(0.2) = 2(3)(0.2) = 1.2 \text{ square inches}$$

3. (a) Marginal cost is $C'(x) = \frac{18x}{9x^2+5}$. Since $9x^2 + 5 > 0$ for all values of x, $\frac{18x}{9x^2+5} > 0$ when $18x > 0$. This occurs when $x > 0$.

(b) If $p = -0.2x + 16$, revenue is given by $R(x) = px = (-0.2x + 16)x = -0.2x^2 + 16x$. Then $R'(x) = -0.4x + 16$, and we have a critical value when $0 = -0.4x + 16$, that is, when $x = 40$. We see

Test Intervals	Test Point	Sign of f'	f incr/decr
$(-\infty, 40)$	20	$+$	increasing
$(40, \infty)$	100	$-$	decreasing

so $x = 40$ will give us an absolute maximum.

(c) Since profit is revenue less cost, we have

$$P(x) = R(x) - C(x) = (-0.2x^2 + 16x) - (5 + 8x) = -0.2x^2 + 8x - 5.$$

Then $P'(x) = -0.4x + 8$, and we have a critical value when $0 = -0.4x + 8$, that is, when $x = 20$. We see

Test Intervals	Test Point	Sign of f'	f incr/decr
$(-\infty, 20)$	10	$+$	increasing
$(20, \infty)$	100	$-$	decreasing

so profit is maximized when $x = 20$. The maximum profit is $P(20) = 75$.

(d) $R'(x) = -3x^2 + 75 = -3(x^2 - 25) = -3(x+5)(x-5)$, so we have critical values when $0 = -3(x+5)(x-5)$, that is, when $x = -5$ or $x = 5$.

Test Intervals	Test Point	Sign of f'	f incr/decr
$(-\infty, -5)$	-100	$-$	decreasing
$(-5, 5)$	0	$+$	increasing
$(5, \infty)$	100	$-$	decreasing

so we have a maximum when $x = 5$.

4. First note, $\frac{dx}{dp} = -\frac{1}{(10+p)^2}$. Then

$$E(p) = -\frac{p}{x}\frac{dx}{dp} = -\frac{p}{\left(\frac{1}{10+p}\right)}\left(-\frac{1}{(10+p)^2}\right) = \frac{p}{10+p}.$$

- $E(5) = 1/3$
- $E(10) = 1/2$
- $E(100) = 10/11$

In each of these cases $E < 1$, so demand is inelastic.

5. First note, $\frac{dx}{dp} = -\frac{3}{p^4}$. Then

$$E(p) = -\frac{p}{x}\frac{dx}{dp} = -\frac{p}{\left(\frac{1}{p^3}\right)}\left(\frac{3}{p^4}\right) = 3.$$

Since $E(p) = 3$ for all p, there are no values of p that will make $E = 1$. For the last part of the problem, we use the theorem that says revenue is maximized when $E = 1$. But since there is no p which will make $E = 1$, for this problem, there is no solution.

6. (a) Critical values of f: $x = 2$ and $x = 4$

Test Intervals	Test Point	Sign of $f'(x)$	f incr/decr
$(-\infty, 2)$	0	$+$	increasing
$(2, 4)$	3	$-$	decreasing
$(4, \infty)$	10	$+$	increasing

f is increasing on $(-\infty, 2) \cup (4, \infty)$ and decreasing on $(2, 4)$

(b) Critical values of f: $x = -1$, $x = 1$, and $x = 2$

Test Intervals	Test Point	Sign of $f'(x)$	f incr/decr
$(-\infty, -1)$	-10	$-$	decreasing
$(-1, 1)$	0	$-$	decreasing
$(1, 2)$	$\frac{3}{2}$	$-$	decreasing
$(2, \infty)$	5	$+$	increasing

f is increasing on $(2, \infty)$ and decreasing on $(-\infty, -1) \cup (-1, 1) \cup (1, 2)$
(also acceptable: f is increasing on $(2, \infty)$ and f is decreasing on $(-\infty, 2)$)

(c) Critical values of f: $x = 1$, $x = -2$

Test Intervals	Test Point	Sign of $f'(x)$	f incr/decr
$(-\infty, -2)$	-10	$+$	increasing
$(-2, 1)$	0	$-$	decreasing
$(1, 3)$	2	$+$	increasing
$(3, \infty)$	10	$+$	increasing

f is increasing on $(-\infty, -2) \cup (1, 3) \cup (3, \infty)$ and decreasing on $(-2, 1)$

7. (a) $f'(x) = -24x^5 - 20x^3 = -4x^3(6x^2 + 5)$, so we have only one critical value $x = 0$.

Test Intervals	Test Point	Sign of $f'(x)$	f incr/decr
$(-\infty, 0)$	-100	$+$	increasing
$(0, \infty)$	100	$-$	decreasing

so we have a relative maximum of $f(0) = 5$

(b) First, let us observe that the domain of g is $(0, \infty)$. Next, observe $g'(x) = 1 - 1/x = \frac{x-1}{x}$, so we have only one critical value $x = 1$.

Test Intervals	Test Point	Sign of $g'(x)$	g incr/decr
$(0, 1)$	$\frac{1}{2}$	$-$	decreasing
$(1, \infty)$	100	$+$	increasing

so we have a relative minimum of $g(1) = 1$.

8. • The critical values of $f(x)$ are the x-values that are in the domain of f where $f'(x) = 0$ or $f'(x)$ is undefined. According to the graph, $f'(x)$ is defined everywhere and $f'(x) = 0$ when $x = -4$, $x = -1$, $x = 2$) and $x = 3$.

- f is increasing when $f'(x) > 0$. This occurs on $(-4,-1) \cup (-1,2) \cup (3,\infty)$. Also acceptable: $(-4,2) \cup (3,\infty)$
- f is decreasing when $f'(x) < 0$. This occurs on $(-\infty,-4) \cup (2,3)$.
- The x-values of local maxima are $x = 2$
- The x-values of local minima are $x = -4$, $x = 3$
- The graph of f is concave up when $f'(x)$ is increasing. This occurs on $(-\infty,-3.25) \cup (-1,0.75) \cup (2.6,\infty)$ (decimals are approximate)
- The graph of f is concave down when $f'(x)$ is decreasing. This occurs on $(-3.25,-1) \cup (0.75,2.6)$
- The inflection values are the x-values on which the graph of f changes concavity. This occurs when $x = -3.25$, $x = -1$, $x = 0.75$, and $x = 2.6$

9. (a)

Test Intervals	Test Point	Sign of $f''(x)$	f concave up/down
$(-\infty,0)$	-10	$+$	concave up
$(0,2)$	1	$-$	concave down
$(2,4)$	3	$-$	concave down
$(4,\infty)$	10	$+$	concave up

f is concave down on $(0,2) \cup (2,4)$ (also acceptable $(0,4)$) and concave up on $(-\infty,0) \cup (4,\infty)$, with inflection values at $x = 0$ and $x = 4$

(b)

Test Intervals	Test Point	Sign of $f''(x)$	f concave up/down
$(-\infty,-2)$	-100	$-$	concave down
$(-2,0)$	-1	$+$	concave up
$(0,2)$	1	$-$	concave down
$(2,\infty)$	100	$+$	concave up

f is concave up on $(-2,0) \cup (2,\infty)$ and concave down on $(-\infty,-2) \cup (0,2)$ with inflection values at $x = -2, 0, 2$

(c)

Test Intervals	Test Point	Sign of $f''(x)$	f concave up/down
$(-\infty,-2)$	-100	$-$	concave down
$(-2,-1)$	-3/2	$+$	concave up
$(-1,0)$	-1/2	$+$	concave up
$(0,1)$	1/2	$-$	concave down
$(1,3)$	2	$-$	concave down
$(3,\infty)$	100	$+$	concave up

f is concave down on $(-\infty,-2)\cup(0,1)\cup(1,3)$ and concave up on $(-2,-1)\cup(-1,0)\cup(3,\infty)$ with inflection values $x=-2,0,3$

10. (a) $f'(x) = -3x^2+12x-9 \Rightarrow f''(x) = -6x+12 = -6(x-2)$; $f''(x)=0$ when $x=2$

Test Intervals	Test Point	Sign of $f''(x)$	f concave up/down
$(-\infty,2)$	0	$+$	concave up
$(2,\infty)$	100	$-$	concave down

Thus f is concave up on $(-\infty,2)$ and concave down on $(2,\infty)$, with an inflection value at $x=2$.

(b) $g'(x) = xe^x+e^x \Rightarrow g''(x) = (xe^x+e^x)+e^x = xe^x+2e^x = e^x(x+2)$; $g''(x)=0$ when $e^x=0$ or $x+2=0$. Since $e^x>0$ for all values of x, the $g''(x)=0$ only when $x=-2$.

Test Intervals	Test Point	Sign of $g''(x)$	g concave up/down
$(-\infty,-2)$	-10	$-$	concave down
$(-2,\infty)$	0	$+$	concave up

11. *Note: What happens to the graph of f'' is unclear when $x=5$. If you assume that f'' continues to increase, which we will assume here, then $f''(5)=0$ and $f''>0$ on $(5,\infty)$ If you assume differently, your answer will differ from that below. On the exam, we will likely only ask about the domain of the graph that is visible.*

- The graph of f is concave up when $f''>0$. This occurs on $(-\infty,-3)\cup(-1,2)\cup(5,\infty)$
- The graph of f is concave down when $f''<0$. This occurs on $(-3,-1)\cup(2,5)$
- The inflection values are the x-values for which f changes from concave up to concave down or vice-versa. This occurs when $x=-3$, $x=-1$, $x=2$, and $x=5$.

12. (a) $\lim_{x\to\infty}\frac{x^2}{x^3+1} = \lim_{x\to\infty}\frac{x^2}{x^3+1}\left(\frac{1/x^3}{1/x^3}\right) = \lim_{x\to\infty}\frac{1/x}{1+1/x^3} = \frac{0}{1+0} = 0$

(b) $\lim_{x\to\infty}\sqrt{x} = \infty$

(c) $\lim_{x\to-\infty}\frac{x^4-x^2+x-1}{x^3+1} = \lim_{x\to-\infty}\frac{x^4-x^2+x-1}{x^3+1}\left(\frac{1/x^3}{1/x^3}\right) = \lim_{x\to-\infty}\frac{x-1/x+1/x^2-1/x^3}{1+1/x^3} = -\infty$

(d) $\lim_{x\to\infty}\frac{x^2-1}{\sqrt{x}-3} = \lim_{x\to\infty}\frac{x^2-1}{\sqrt{x}-3}\left(\frac{1/\sqrt{x}}{1/\sqrt{x}})\right) = \lim_{x\to\infty}\frac{x^{3/2}-1/\sqrt{x}}{1-3/\sqrt{x}} = \infty$

(e) $\lim_{x\to-\infty}\sqrt{1-\frac{1}{x^2}} = 1$, since $\lim_{x\to\infty}1/x^2 = 0$

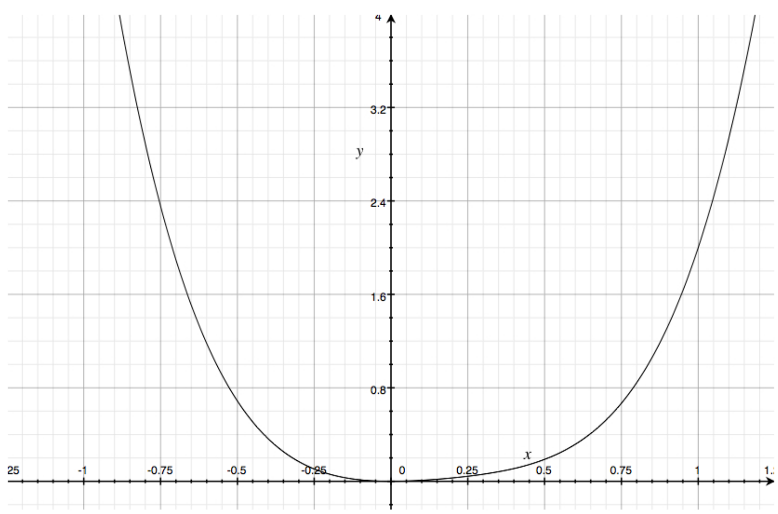
(f) $\lim_{x\to\infty}\frac{6e^x}{e^x-2e^{-x}} = \lim_{x\to\infty}\frac{6e^x}{e^x-2e^{-x}}\left(\frac{1/e^x}{1/e^x}\right) = \lim_{x\to\infty}\frac{6}{1-2e^{-2x}} = 6$

13.
- the critical values of $f(x)$: $x = -4, -2.3, 0.85, 3$ (values are approximate)
- the largest open intervals on which $f(x)$ is increasing: $(-\infty, -4) \cup (-2.3, 0.85) \cup (3, \infty)$
- the largest open intervals on which $f(x)$ is decreasing: $(-4, -2.3) \cup (0.85, 3)$
- the x-values of all relative maxima: $x = -4, 0.85$
- the x-values of all relative minima: $x = -2.3, 3$
- all inflection values: $x = -3, -0.5, 2$ (values are approximate)
- the largest open intervals on which the graph of f is concave up: $(-3, -0.5) \cup (2, \infty)$
- the largest open intervals on which the graph of f is concave down: $(-\infty, -3) \cup (-0.5, 2)$
- any horizontal or vertical asymptotes: none

14. (a)
- Domain: $(-\infty, \infty)$
- $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = \infty$, so there are no horizontal asymptotes
- There are no vertical asymptotes
- $f'(x) = 12x^3 - 6x^2 + 2x = 2x(6x^2 - 3x + 1)$, so we have a critical value when $x = 0$

Test Intervals	Test Point	Sign of f'	f incr/decr
$(-\infty, 0)$	-1	$-$	decreasing
$(0, \infty)$	1	$+$	increasing

- $f''(x) = 36x^2 - 12x + 2 = 2(18x^2 - 6x + 1)$ does not equal 0 for any value of x. Since $f''(x) > 0$ for all values of x, the function is concave up on $(-\infty, \infty)$



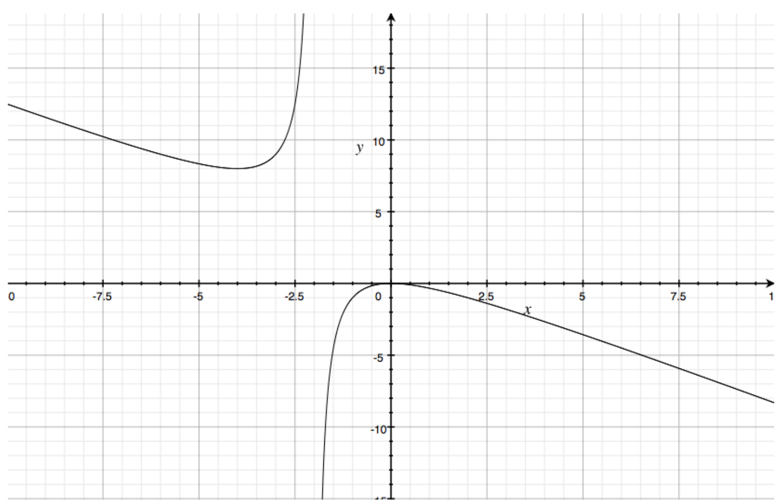
(b)

- Domain: $(-\infty, -2) \cup (-2, \infty)$
- $\lim\limits_{x\to\infty} -\frac{x^2}{x+2} = \lim\limits_{x\to\infty} -\frac{x^2}{x+2}\left(\frac{1/x}{1/x}\right) = \lim\limits_{x\to\infty} -\frac{x}{1+2/x} = -\infty$
  $\lim\limits_{x\to-\infty} -\frac{x^2}{x+2} = \lim\limits_{x\to-\infty} -\frac{x^2}{x+2}\left(\frac{1/x}{1/x}\right) = \lim\limits_{x\to-\infty} -\frac{x}{1+2/x} = \infty$
  There are no horizontal asymptotes
- $\lim\limits_{x\to-2^-} -\frac{x^2}{x+2} = \infty$
  $\lim\limits_{x\to-2^+} -\frac{x^2}{x+2} = -\infty$
  $x = 2$ is a vertical asymptote
- $g'(x) = -\frac{x(x+4)}{(x+2)^2}$; $g'(x) = 0$ when $x = 0$ or $x = -4$; $g'(x)$ is undefined when $x = -2$

Test Intervals	Test Point	Sign of g'	g incr/decr
$(-\infty, -4)$	-5	$-$	decreasing
$(-4, -2)$	-3	$+$	increasing
$(-2, 0)$	-1	$+$	increasing
$(0, \infty)$	2	$-$	decreasing

- $g''(x) = -\frac{8}{(x+2)^3}$; $g''(x) \neq 0$ for any values of x; $g''(x)$ is undefined when $x = -2$

Test Intervals	Test Point	Sign of g''	g concave up/down
$(-\infty, -2)$	-5	$+$	concave up
$(-2, \infty)$	0	$-$	concave down



(c)

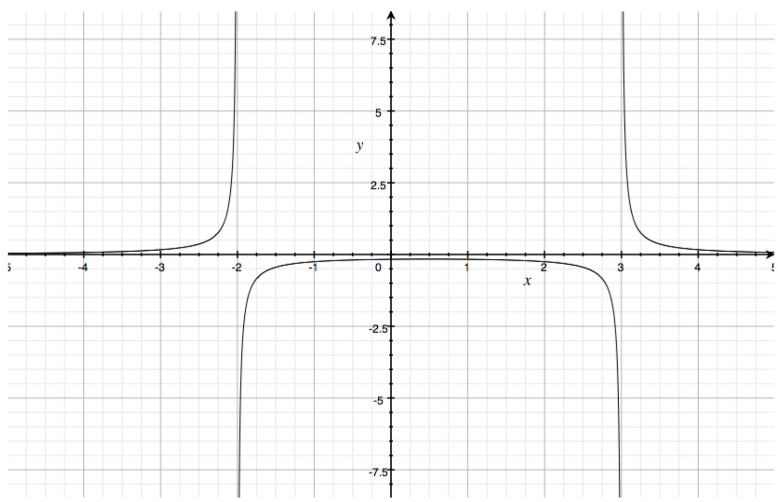
- Observe the denominator is 0, that is $x^2 - x - 6 = (x-3)(x+2) = 0$,when $x = 3$ and $x = -2$. Thus the domain of the function is $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
- $\lim\limits_{x\to-\infty} \frac{1}{x^2-x-6} = \lim\limits_{x\to-\infty} \frac{1}{x^2-x-6}\left(\frac{1/x^2}{1/x^2}\right) = \frac{1/x^2}{1-1/x-6/x^2} = 0$; similarly, $\lim\limits_{x\to\infty} h(x) = 0$
  y=0 is a horizontal asymptote

- $\lim\limits_{x\to -2^-} h(x) = \infty$; $\lim\limits_{x\to -2^+} h(x) = -\infty$; $\lim\limits_{x\to 3^-} h(x) = -\infty$; $\lim\limits_{x\to 3^+} h(x) = \infty$
  There are vertical asymptotes at $x = -2$ and $x = 3$
- $h'(x) = \frac{1-2x}{(x^2-x-6)^2}$; $h'(x) = 0$ when $x = 1/2$, and $h'(x)$ is not defined when $x = -2$ or $x = 3$

Test Intervals	Test Point	Sign of h'	h incr/decr
$(-\infty, -2)$	-5	$+$	increasing
$(-2, 1/2)$	0	$+$	increasing
$(1/2, 3)$	1	$-$	decreasing
$(3, \infty)$	5	$-$	decreasing

- $h''(x) = \frac{2(3x^2-3x+7)}{(x^2-x-6)^3}$; $h''(x) \neq 0$ for any value of x since $3x^2 - 3x + 7 > 0$ for all x, and $h''(x)$ is not defined when $x = -2$ or $x = 3$

Test Intervals	Test Point	Sign of h''	h concave up/down
$(-\infty, -2)$	-5	$+$	concave up
$(-2, 3)$	0	$-$	concave down
$(3, \infty)$	10	$+$	concave up



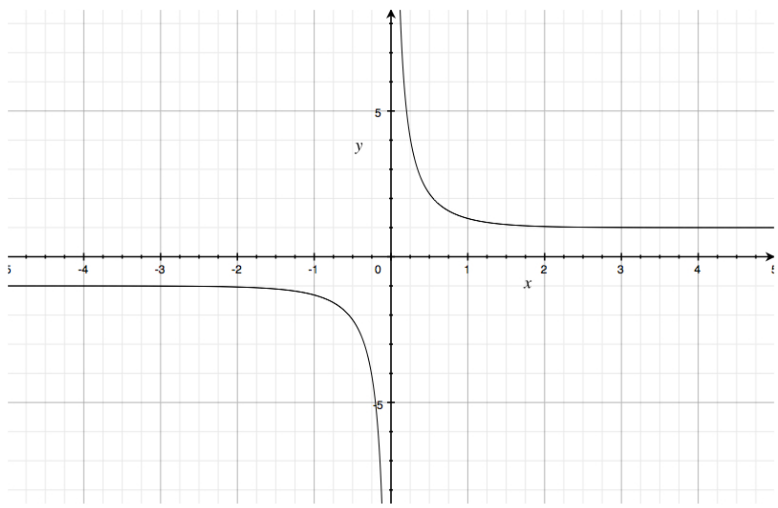
(d)
- Observe $e^x - e^{-x} = 0$ when $x = 0$, so the domain of j is $(-\infty, 0) \cup (0, \infty)$.
- $\lim\limits_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} = \lim\limits_{x\to\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} \left(\frac{1/e^x}{1/e^x}\right) = \lim\limits_{x\to\infty} \frac{e^x/e^x+e^{-x}/e^x}{e^x/e^x-e^{-x}/e^x} = \lim\limits_{x\to\infty} \frac{1+e^{-2x}}{1-e^{-2x}} = \frac{1+0}{1-0} = 1$
  $\lim\limits_{x\to-\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} = \lim\limits_{x\to-\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} \left(\frac{1/e^{-x}}{1/e^{-x}}\right) = \lim\limits_{x\to-\infty} \frac{e^x/e^{-x}+e^{-x}/e^{-x}}{e^x/e^{-x}-e^{-x}/e^{-x}} = \lim\limits_{x\to-\infty} \frac{e^{2x}+1}{e^{2x}-} = \frac{1+0}{1-0} = -1$
  There are horizontal asymptotes at $y = 1$ and $y = -1$.
- $\lim\limits_{x\to 0^-} \frac{e^x+e^{-x}}{e^x-e^{-x}} = -\infty$ and $\lim\limits_{x\to 0^+} \frac{e^x+e^{-x}}{e^x-e^{-x}} = \infty$
  There is a vertical asymptote at $x = 0$

- $j'(x) = \frac{-4}{(e^x - e^{-x})^2}$; $j'(x) = 0$ for no value of x, so there are no critical points.

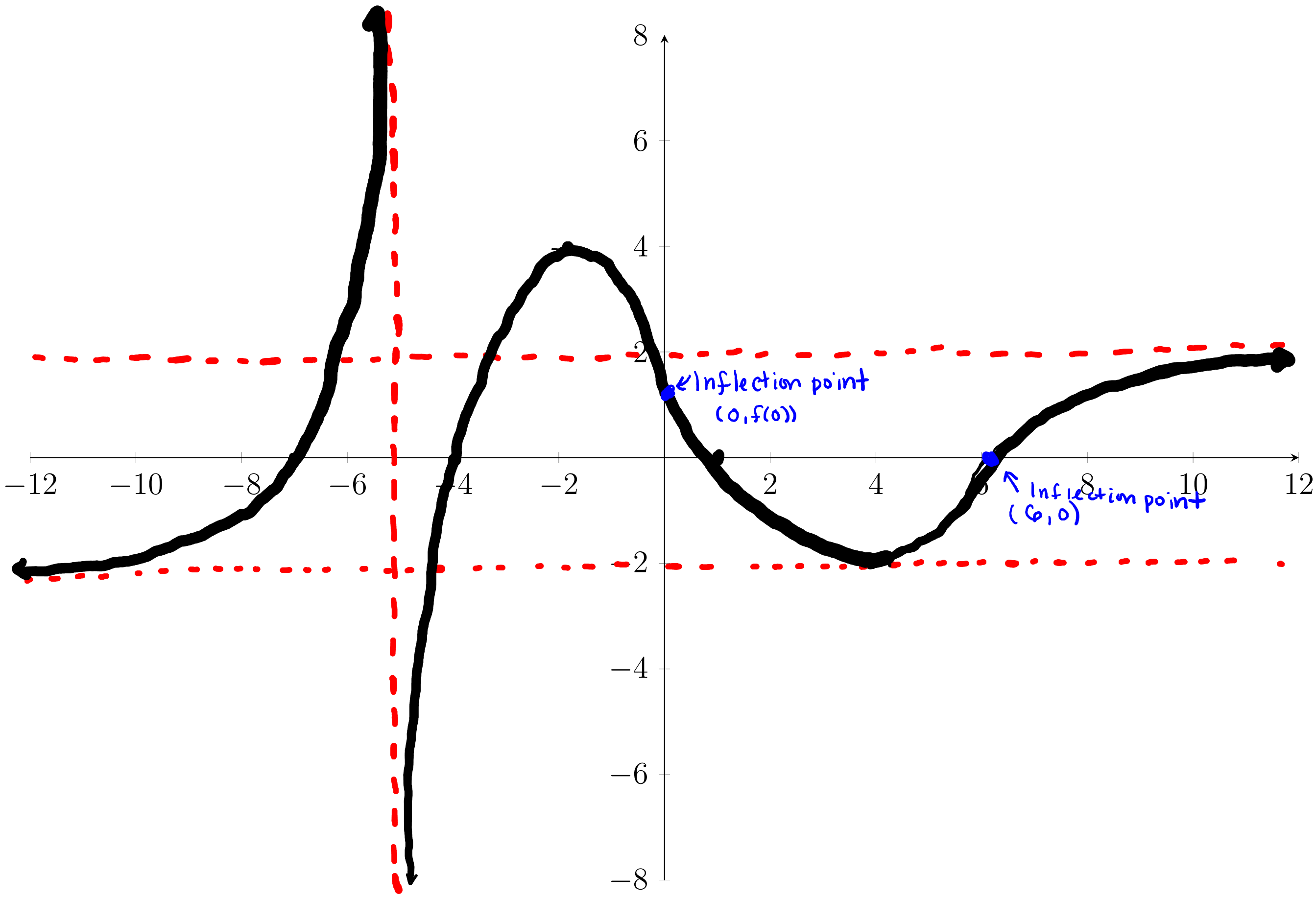
Test Intervals	Test Point	Sign of j'	j incr/decr
$(-\infty, 0)$	-10	$-$	decreasing
$(0, \infty)$	10	$-$	decreasing

- $j''(x) = \frac{8(e^x + e^{-x})}{(e^x - e^{-x})^3}$; $j''(x) = 0$ for no values of x

Test Intervals	Test Point	Sign of j''	j concave up/down
$(-\infty, 0)$	-10	$-$	concave down
$(0, \infty)$	10	$+$	concave up



15. The graph of $f(x)$ is on the next page



16. (a) $f'(x) = -6x(x-1)$, so the critical values of f are $x = 0$ and $x = 1$

- On $[-\frac{1}{2}, \frac{1}{2}]$, $f(-1/2) = 1$, $f(0) = 0$, $f(1/2) = 1/2$, so the absolute maximum is $f(-1/2) = 1$ and the absolute minimum is $f(0) = 0$
- On $(-\frac{1}{2}, \frac{1}{2})$, we no longer have an absolute maximum. However, the absolute minimum is still $f(0) = 0$
- On $[0, 2]$, $f(0) = 0$, $f(1) = 1$ and $f(2) = -4$, so the absolute maximum is $f(1) = 1$ and the absolute minimum is $f(2) = -4$
- On $[0, 2)$, we no longer have an absolute minimum. However, the absolute maximum is still $f(1) = 1$

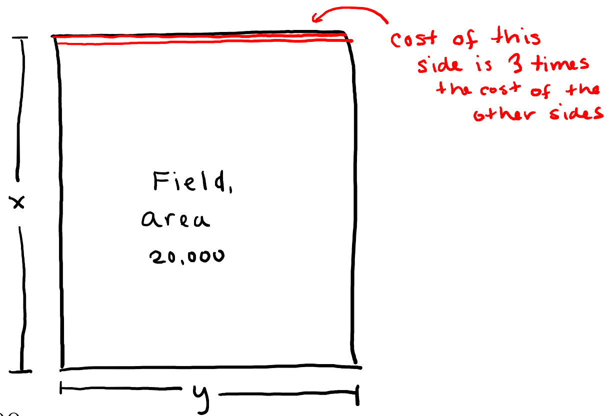
(b) $f'(x) = x^2(5x^2 - 16x + 3)$, so the critical values are $x = 0$, $x = 1/5$, and $x = 3$

- On $[0, 2]$, $f(0) = -10$, $f(1/5) = -9.99808$, $f(2) = -34$, so the absolute maximum is $f(1/5) = -9.99808$ and the absolute minimum is $f(2) = -34$
- On $[0, 4]$, $f(0) = -10$, $f(1/5) = -9.99808$, $f(3) = -64$, $f(4) = 54$, so the absolute maximum is $f(4) = 54$ and the absolute minimum is $f(3) = -64$

- On $(-\infty, \infty)$, we observe that $\lim\limits_{x\to-\infty} x^5 - 4x^4 + x^3 - 10 = -\infty$ and $\lim\limits_{x\to\infty} x^5 - 4x^4 + x^3 - 10 = \infty$, so there is no absolute maximum or minimum

(c) $f'(x) = 4x^3 + 6x = 2x(2x^2 + 3)$ has a critical value at $x = 0$. Observe that on $(-\infty, 0)$, $f'(x) < 0$ and on (0∞), $f'(x) > 0$, so $f(0) = -1$ is an absolute minimum. Moreover, since $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = \infty$, there is no absolute maximum.

17. Let x denote the length of the field and y denote the width of the field.



Then $xy = 20,000$.

The cost per foot of three of the sides is C, where $C > 0$, and the cost of the fourth side is $3C$. Then the total cost of the fence is given by

$$3Cy + Cx + Cy + Cx = 4Cy + 2Cx.$$

Since $xy = 20,000$, we have $x = 20,000/y$. Substituting this into the above equation gives a cost equation (in terms of y):

$$f(y) = 4Cy + \frac{40,000C}{y}.$$

We want to minimize this function on the interval $(0, \infty)$ (since we want the width of the field to be positive).

Then

$$f'(y) = 4C - 40,000Cy^{-2} = 4C - \frac{40,000C}{y^2} = \frac{4Cy^2 - 40,000C}{y^2}.$$

The critical value of f occurs when $4Cy^2 - 40,000C = 0$, so $y = \pm 100$, but only $y = 100$ makes sense for this problem.

To see that this is indeed minimum, we can either use the first or second derivative test (you should choose which one you like better and show your work)

18. Let x denote the number of additional trees over 1000. (So, for example if we plant 1001 trees, $x = 1$, if we plant 1002 trees, $x = 2$ and so on.)

    Then the total revenue is given by

$$(50 - 0.02x)(1000 + x)$$

and the total cost of maintenance is given by

$$10(1000 + x).$$

Thus the total profit from the orchard is given by revenue less cost, or

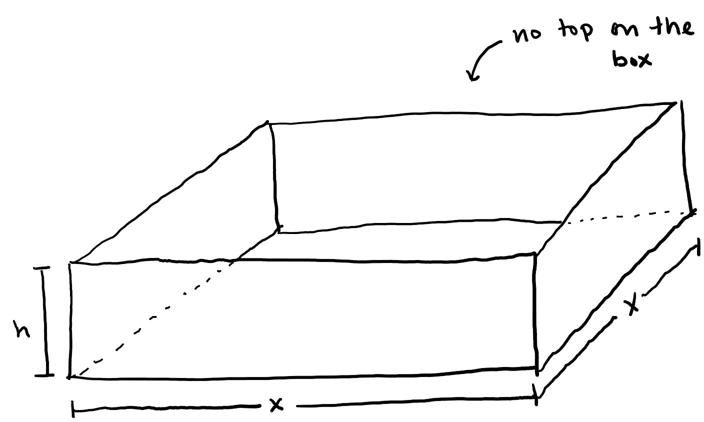
$$f(x) = (50 - 0.02x)(1000 + x) - 10(1000 + x) = -0.02x^2 + 20x + 40,000.$$

To find the maximum profit, we look to the derivative:

$$f'(x) = 20 - 0.04x.$$

The critical values of the derivative occurs when $20 - 0.04x = 0$, or when $x = 500$. Observe that $f''(x) = -0.04$, so $f''(500) = -0.04 < 0$. This implies $x = 500$ is a maximum. Hence we should plant 1500 trees to maximize profit for the orchard.

19. Since the box has a square base, denote the length of each side of the base by x, and the height of the box by h.



Then the volume of the box is given by

$$x^2h = 27.$$

If the cost per square inch of the side of the box is C, where $C > 0$, then the cost per square inch of the of the bottom of the box is given by $2C$. Then the total cost of the box is given by

$$2Cx^2 + 4Cxh.$$

Since $x^2h = 27$, we have $h = 27/x^2$. Substituting this into the above equation gives a cost equation (in terms of x):

$$f(x) = 2Cx^2 + 4Cx\left(\frac{27}{x^2}\right) = 2Cx^2 + \frac{108C}{x} = \frac{2Cx^3 + 108C}{x}.$$

We want to minimize this function on the interval $(0, \infty)$ (since we want the box to have a positive side length).

Then

$$f'(x) = \frac{4C(x^3 - 27)}{x^2}.$$

The critical value of f occurs when $x^3 - 27 = 0$, or when $x = 3$. Observe $f'(x) < 0$ on $(0, 3)$ and $f'(x) > 0$ on $(3, \infty)$, so $x = 3$ is an absolute minimum.

Therefore, the box should have dimensions $3''x3''x3''$ to minimize cost. The minimum cost is $54C$, where C is the cost per square inch of material used.