Math 1071 Spring 2016, Exam 1 Review Answers

1. (a) $(-\infty, \infty)$

   (b) $x + 1 \geq 0$ which implies $x \geq -1$. Thus the domain is $[-1, \infty)$

   (c) Since the denominator of $h(x)$ cannot be zero, we see $h(x)$ is undefined when $x^2 - 1 = 0$. Hence the domain is all real numbers except $x = 1$ and $x = -1$. In interval notation, this is given by $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

   (d) Since the denominator of $k(x)$ is only defined and makes sense (i.e. is not 0) when $x - 2 > 0$, we must have $x > 2$. Hence the domain in interval notation is given by $(2, \infty)$.

   (e) We can only take the natural log of positive numbers, so $2x + 3 > 0$. This implies the domain is $x > -\frac{3}{2}$. Thus the domain in interval notation is given by $(-3/2, \infty)$

2. Solve the following equations for x:

   (a)

$$\begin{aligned}
\frac{1}{3^{-x}} \cdot 9^{x+1} = 1 &\Rightarrow 3^x \cdot (3^2)^{x+1} = 1 \\
&\Rightarrow 3^x \cdot 3^{2x+2} = 1 \\
&\Rightarrow 3^{3x+2} = 3^0 \\
&\Rightarrow 3x + 2 = 0 \\
&\Rightarrow x = -2/3
\end{aligned}$$

(b)

$$
\begin{aligned}
4^{2x} = 8^{9x+15} &\Rightarrow (2^2)^{2x} = (2^3)^{9x+15} \\
&\Rightarrow 2^{4x} = 2^{27x+45} \\
&\Rightarrow 4x = 27x + 45 \\
&\Rightarrow 45 = 23x \\
&\Rightarrow x = -45/23
\end{aligned}
$$

(c)

$$
\begin{aligned}
5^{2x-1} \cdot 5 = \frac{1}{5^x} &\Rightarrow 5^{2x-1} \cdot 5 = 5^{-x} \\
&\Rightarrow 5^{2x} = 5^{-x} \\
&\Rightarrow 2x = -x \\
&\Rightarrow 3x = 0 \\
&\Rightarrow x = 0
\end{aligned}
$$

(d)

$$
\begin{aligned}
7 \cdot 3^{2x+4} - 1 = 0 &\Rightarrow 7 \cdot 3^{2x+4} = 1 \\
&\Rightarrow 3^{2x+4} = \frac{1}{7} \\
&\Rightarrow \log_3(3^{2x+4}) = \log_3 \frac{1}{7} \\
&\Rightarrow 2x + 4 = \log_3 \frac{1}{7} \\
&\Rightarrow 2x = \log_3 \frac{1}{7} - 4 \\
&\Rightarrow x = \frac{\log_3(1/7) - 4}{2}
\end{aligned}
$$

(e)

$$
\begin{aligned}
\log_2(2x-2) - \log_2(x-1) = 0 &\Rightarrow \log_2(2x-2) = \log_2(x-1) \\
&\Rightarrow 2x - 2 = x - 1 \\
&\Rightarrow x = 1
\end{aligned}
$$

Observe, however, that when $x = 1$, $2(1) - 2 = 0$ and $1 - 1 = 0$, and we cannot take a logarithm (with any base!) of 0. Thus there are no solutions.

(f)

$$\begin{aligned}\ln(x^2+2)-\ln(3x)=0 &\Rightarrow \ln(x^2+2)=\ln(3x)\\ &\Rightarrow x^2+2=3x\\ &\Rightarrow x^2-3x+2=0\\ &\Rightarrow (x-2)(x-1)=0\\ &\Rightarrow x=2, x=1\end{aligned}$$

(Here, both values of x are valid solutions)

3.

$$P=\frac{1800}{\left(1+\frac{.036}{4}\right)^{(4)(1.5)}}\approx \$17,057.90$$

4. (a) The maximal revenue is the y value of the vertex of the graph of $R(x)$. To find the y-value, we first find the x-value of the vertex:

$$x=\frac{-b}{2a}=\frac{-20}{-2(-2)}=5$$

(this gives the number of items that must be sold to obtain the maximal revenue). The maximal revenue itself is given by

$$R(5)=-2(5^2)+20(5)=-50+100=50.$$

Thus the maximal revenue is \$50.

(b) The break even quantity is the x value for which $C(x)=R(x)$:

$$2x+10=-2x^2+20x.$$

That is, $2x^2-18x+10=0$. Solving for x using the quadratic formula gives $x=\frac{9+\sqrt{61}}{2}$ and $x=\frac{9-\sqrt{61}}{2}$

5. Let t denote time (in months) from which Sadie purchased her laptop. Then at $t=0$, when the laptop was first purchased, $V=\$1800.00$. This gives a point of the form $(t,V)=(0,1800.00)$. Since 6 months have passed, and Sadie estimates that her laptop will

have a value of $V = \$1382.00$ 5 months from now, this gives a point of the form $(t, V) = (11, 1382.00)$. Using this as points that lie on the line which describes depreciation, we have

$$m = \frac{1382 - 1800}{11 - 0} = \frac{-418}{11} = -38.$$

Thus our linear depreciation model is given by

$$V - 1800 = -38(x - 0).$$

6. (a) Using the fact that profit is revenue less cost, where $R(x) = 190x$ and $C(x) = 55x + 18500$, we have

$$P(x) = 135x - 18500$$

   (b) Break even quantity occurs is the x value for which $P(x) = 0$, that is, when $135x - 18500 = 0$. Solving for x gives that the break even quantity is $x = 18500/135 \approx 137$ guitars.

   (c) The break even *revenue* is $R(137) = \$26030$

7. Let P denote the amount invested in our account now. If we want our investment to double, in t years from now, we want $F = 2P$. Then

$$2P = Pe^{.1t} \Rightarrow 2 = e^{0.1t} \Rightarrow \ln 2 = 0.1t \Rightarrow t = \frac{\ln 2}{0.1} \approx 6.93 \text{ years.}$$

8. (a)

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 8x + 12} = \lim_{x\to 2} \frac{(x+3)(x-2)}{(x-6)(x-2)} = \lim_{x\to 2} \frac{(x+3)}{(x-6)} = -\frac{5}{4}$$

   (b) Since $f(x) = \frac{x^2+x-6}{x^2-8x}$ is continuous at $x = 2$,

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x^2 - 8x} = \frac{(2)^2 + 2 - 6}{(2)^2 - 8(2)} = \frac{0}{-12} = 0$$

   (c)

$$\lim_{x\to 2} \frac{x^2 + x}{x^2 - 8x + 12} = \lim_{x\to 2} \frac{x(x+1)}{(x-6)(x-2)}$$

   Since no terms cancel out, we cannot evaluate this limit algebraically. Notice that when x approaches 2, the numerator approaches 6 and the denominator approaches 0. That is, the numerator gets closer and closer to 6 and the denominator gets closer and closer to 0. This means that the ratio itself will get large without bound since something close to 6 divided by a very, very small number can get very, very big. So the limit does not exist.

9. (a) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{6x-6}{x^2-1} = \lim_{x\to 1^+} \frac{6(x-1)}{(x+1)(x-1)} = \lim_{x\to 1^+} \frac{6}{x+1} = 3.$

   (b) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 - x + 3 = 3.$

   (c) $\lim_{x\to 1} f(x) = 3.$

   (d) First observe that when $x < 1$, $f(x) = x^2 - x + 3$ which is continuous everywhere, and when $x > 1$, $f(x) = \frac{6x-6}{x^2-1}$ which is only discontinuous when $x = -1$ and $x = 1$ (neither of which occur when $x > 1$). Thus when $x \neq 1$, the function is continuous. At $x = 1$, there is a discontinuity since $\lim_{x\to 1} f(x) = 3$ but $f(1) = 3$. Thus the function is continuous on $(-\infty, 1) \cup (1, \infty)$.

10. For each of the following, we use the average rate of change formula:

$$\frac{f(b) - f(a)}{b - a}$$

with $f(x) = 4x^3 - 2x^2 + 7x + 1$

   (a) ($a = 1$, and $b = 4$) 81

   (b) ($a = 1$, and $b = 2$) 29

   (c) ($a = 1$, and $b = 1.5$) 21

   (d) ($a = 1$, and $b = 1.1$) 16.04

   (e) ($a = 1$, and $b = 1.01$) 15.1004

11. Use the limit definition of the derivative to find $f'(x)$. Write an equation of the tangent line to the graph of $f(x)$ at the indicated point.

(a)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(5(x+h)^2+7(x+h)-3)-(5x^2+7x-3)}{h} \\
&= \lim_{h\to 0} \frac{5(x^2+2xh+h^2)+7x+7h-3-5x^2-7x+3}{h} \\
&= \lim_{h\to 0} \frac{5x^2+10xh+5h^2+7x+7h-3-5x^2-7x+3}{h} \\
&= \lim_{h\to 0} \frac{10xh+5h^2+7h}{h} \\
&= \lim_{h\to 0} \frac{h(10x+5h+7)}{h} \\
&= \lim_{h\to 0} 10x+5h+7 \\
&= 10x+7
\end{aligned}
$$

Observe $f'(0) = 7$ and $f(0) = -3$, so the line tangent to the curve at $x = 0$ has a slope of 7 and passes through the point (0,-3). Thus the equation of the tangent line is

$$y-(-3) = 7(x-0) \text{ or } y = 7x-3$$

(b)

$$
\begin{aligned}
\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} \frac{\frac{9}{(x+h)-3}-\frac{9}{x-3}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{9(x-3)-9(x+h-3)}{(x+h-3)(x-3)}\right) \qquad \text{(get a common denominator)} \\
&= \lim_{h\to 0} \frac{-9h}{h(x+h-3)(x-3)} \\
&= \lim_{h\to 0} \frac{-9}{(x+h-3)(x-3)} \\
&= \frac{-9}{(x-3)^2}
\end{aligned}
$$

Observe, $f'(1) = -9/4$ and $f(1) = -9/2$, so the line tangent to the curve at $x = 1$ has a slope of -9/4 and passes through the point (1, -9/2). Thus the equation of the tangent line is

$$y - \left(-\frac{9}{2}\right) = -\frac{9}{4}(x-1) \text{ or } y = -\frac{9}{4}x - \frac{9}{4}$$

(c)

$$\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} \\
&= \lim_{h\to 0}\frac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} \qquad \text{(multiply by the conjugate)} \\
&= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x})} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

Observe $f'(4) = 1/4$ and $f(4) = 2$, so the line tangent to the curve at $x = 4$ has a slope of 1/4 and passes through the point (2,4). Thus the equation of the tangent line is

$$y - 2 = \frac{1}{4}(x-4) \text{ or } y = \frac{1}{4}x + 1$$

12. (a) $f'(x) = -14x + 5$
    (b) $g'(x) = \frac{15}{x} + 7e^x$
    (c) $h'(x) = \frac{1}{3} + \frac{3}{2\sqrt{x}} + \frac{3}{2x^{3/2}} - \frac{10}{3x^{5/2}}$
    (d) $g(x) = \frac{1}{x} + 1 + 4x$ where $x \neq 0$, so $g'(x) = -\frac{1}{x^2} + 4$

13. Find the derivative. You may use whatever rules are appropriate.
    (a) $f'(x) = 5(x^3 - 4x^2 + 1)^4(3x^2 - 8x)$
    (b) $f'(x) = \frac{(x^2+3)^9 e^x - e^x(9(x^2+3)^8(2x))}{(x^2+3)^{18}}$
    (c) $g('x) = x\left(\frac{3x^2-2x}{x^3-x^2+1}\right) + \ln|x^3 - x^2 + 1|$
    (d) $h'(x) = \frac{1}{2}(x^2 + x + 1)^{-1/2}(2x + 1)$