2 i) $f(x) = x^4 - 2x^3 + x^2$

a) find critical points

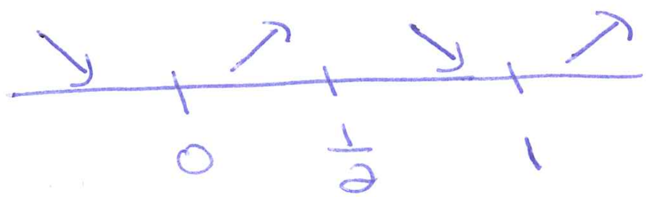
$f' = 4x^3 - 6x^2 + 2x$

$0 = 2x(2x^2 - 3x + 1)$

$0 = 2x(2x - 1)(x - 1)$

$x = 0, 1, \frac{1}{2}$

b) find intervals of incr./decr.



f increasing on $(0, \frac{1}{2}) \cup (1, \infty)$

f decreasing on $(-\infty, 0) \cup (\frac{1}{2}, 1)$

c) find relative extrema

$x = 0, 1$ relative min

$x = \frac{1}{2}$ relative max

d) find intervals of concave up/concave down

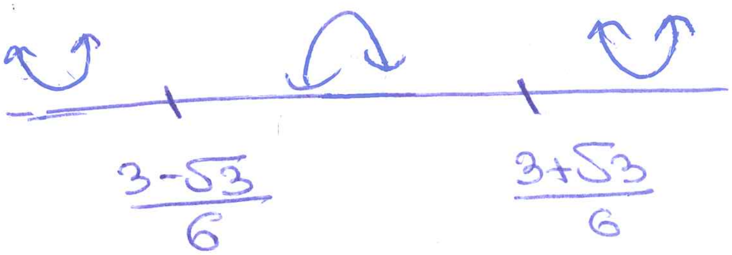
$f'' = 12x^2 - 12x + 2$

$0 = 2(6x^2 - 6x + 1)$

use quadratic to solve $6x^2 - 6x + 1 = 0$

$$\frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 6 \cdot 1}}{2 \cdot 6} = \frac{6 \pm \sqrt{12}}{12} = \frac{6 \pm 2\sqrt{3}}{12}$$

$$= \frac{3 \pm \sqrt{3}}{6}$$



f is concave down on $\left(\frac{3-\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}\right)$

f is concave up on $\left(-\infty, \frac{3-\sqrt{3}}{6}\right) \cup \left(\frac{3+\sqrt{3}}{6}, \infty\right)$

e) find inflection points

$x = \frac{3-\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}$

f) <u>sketch</u> points on the graph

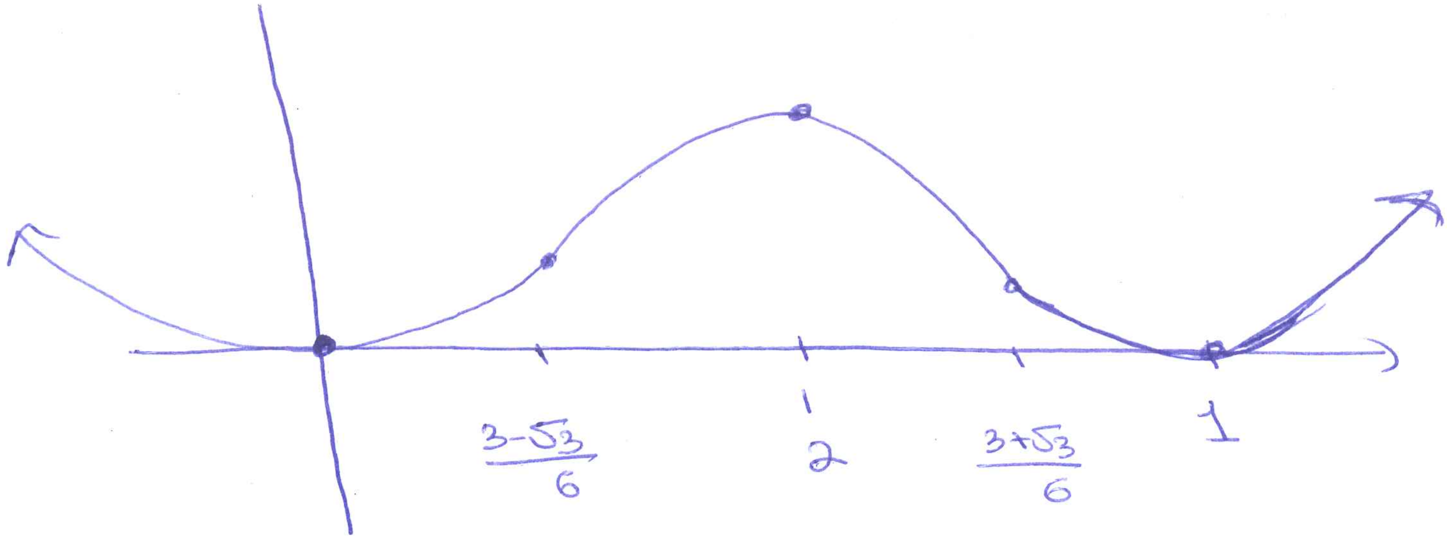
$(0, 0)$

$(1, 0)$

$\left(\frac{1}{2}, \frac{1}{16}\right) = \left(\frac{1}{2}, .0625\right)$

$\left(\frac{3-\sqrt{3}}{6}, \frac{1}{36}\right) \approx (.211, .0278)$

$\left(\frac{3+\sqrt{3}}{6}, \frac{1}{36}\right) \approx (.7887, .0277)$



ii) $f(x) = \dfrac{x+1}{x-1}$

a) domain of $f(x)$ $(-\infty, 1) \cup (1, \infty)$

b) find critical pt.

$$f' = \frac{(1)(x-1) - (1)(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$$0 = \frac{-2}{(x-1)^2}$$ no solution

no critical pt.

c) find intervals of incr/decr

f is decreasing on $(-\infty, 1) \cup (1, \infty)$

d) there are no relative min/max

e) find intervals of concave up/down

$$f' = -2(x-1)^{-2}$$

$$f'' = +4(x-1)^{-3}\cdot(1) = \frac{+4}{(x-1)^3}$$

$$0 = \frac{4}{(x-1)^3}$$ no solution

$\cap$ | 1 | $\cup$

f concave up $(1, \infty)$
f concave down $(-\infty, 1)$

f) no inflection pt. b/c $x=1$ is not in the domain of $f(x)$

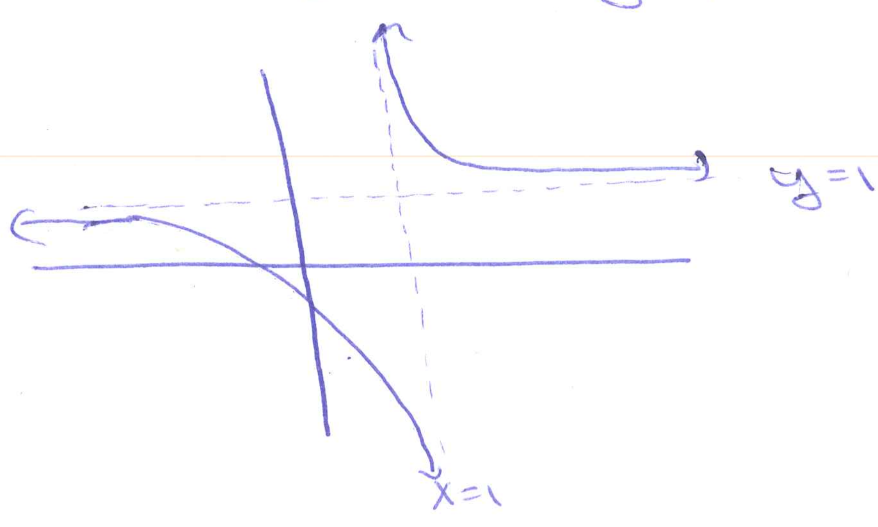
g) horizontal assymptotes

$$\lim_{x \to \infty} \frac{x+1}{x-1} = 1$$

$$\lim_{x \to -\infty} \frac{x+1}{x-1} = 1$$

$y = 1$ is a horizontal assymptote

h) sketch



III) $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

a) observe $e^x - e^{-x} = 0$

$e^x = e^{-x}$

$x = 0$ so domain of f is $(-\infty, 0) \cup (0, \infty)$

b) find critical pt.

$$f' = \frac{(e^x - e^{-x})(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$$

$$= \frac{e^{2x} - \underbrace{e^x e^{-x}}_{1} - \underbrace{e^{-x} \cdot e^x}_{1} + e^{-2x} - e^{2x} - e^x e^{-x} - e^{-x} e^x - e^{-2x}}{(e^x - e^{-x})^2}$$

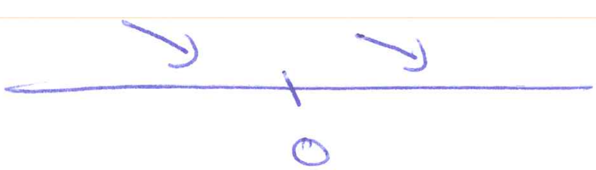
$$= \frac{-4}{(e^x - e^{-x})^2}$$

$$0 = \frac{-4}{(e^x - e^{-x})^2}$$ no solution

no critical pt.

c) intervals of incr. / decr.

f is decreasing on $(-\infty, 0) \cup (0, \infty)$



note no relative extrema

d) find intervals of concave up/ down

$f' = -4(e^x - e^{-x})^{-2}$

$f'' = 8(e^x - e^{-x})^{-3} \cdot (e^x + e^{-x}) = \dfrac{8(e^x + e^{-x})}{(e^x - e^{-x})^3}$

$0 = \dfrac{8(e^x + e^{-x})}{(e^x - e^{-x})^3}$

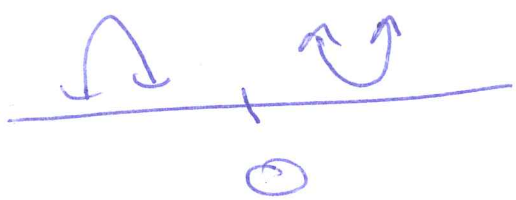
$0 = 8(e^x + e^{-x})$

$0 = e^x + e^{-x}$ no solution b/c

$e^x > 0$

$e^{-x} > 0$ for all x

so they cannot add up to 0.



f concave up $(0, \infty)$

f concave down $(-\infty, 0)$

e) no inflection points

f) horizontal assymptotes

$\lim\limits_{x \to \infty} \dfrac{e^x + e^{-x}}{e^x - e^{-x}} = \lim\limits_{x \to \infty} \dfrac{\frac{e^x}{e^x} + \frac{e^{-x}}{e^x}}{\frac{e^x}{e^x} - \frac{e^{-x}}{e^x}}$

$= \lim\limits_{x \to \infty} \dfrac{1 + e^{-2x}}{1 - e^{-2x}} = \dfrac{1+0}{1-0} = 1$

note $\lim\limits_{x \to \infty} e^{-2x} = 0$

$$\lim_{x\to-\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}} = \lim_{x\to-\infty} \frac{\frac{e^x}{e^{-x}}+\frac{e^{-x}}{e^{-x}}}{\frac{e^x}{e^{-x}}-\frac{e^{-x}}{e^{-x}}} =$$

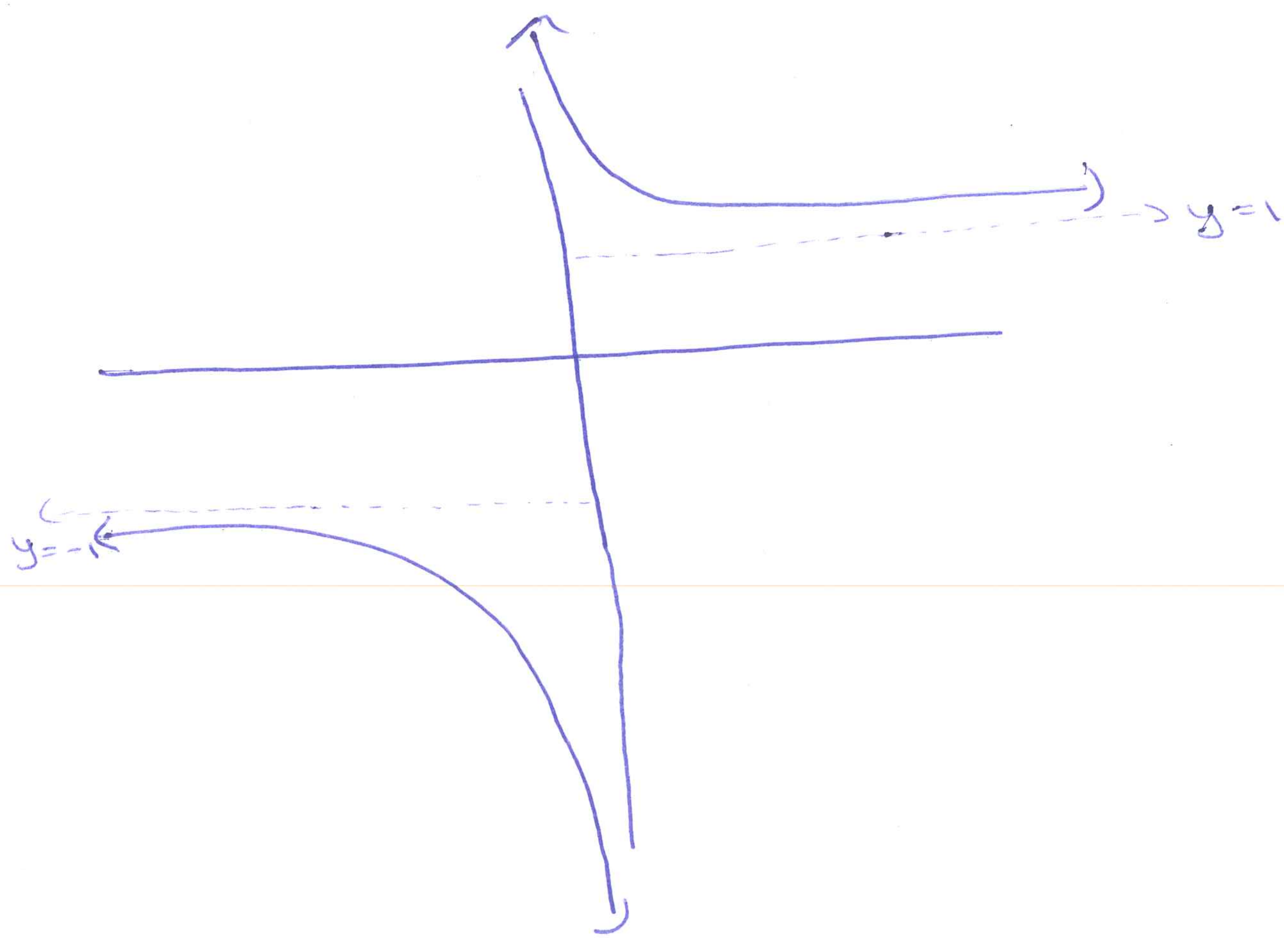
$$= \lim_{x\to-\infty} \frac{e^{2x}+1}{e^{2x}-1} = \frac{0+1}{0-1} = -1$$

<u>note</u> $\lim_{x\to-\infty} e^{2x} = 0$

so we have horizontal assymptotes

$$y = 1, -1$$

g) <u>sketch</u>



#3 i) $p(x) = e^{-2x}$

$R(x) = p \cdot x = xe^{-2x}$

marginal revenue $R'(x) = e^{-2x} + (x)e^{-2x} \cdot (-2)$

$R'(x) = 0 = e^{-2x} - 2xe^{-2x}$

$0 = e^{-2x}(1-2x)$

$e^{-2x} = 0$ no solution

$1 - 2x = 0$

$\boxed{x = \frac{1}{2}}$

ii) $c(x) = \ln(9x^2+5) + 100$

marginal cost $c'(x) = \frac{1}{9x^2+5} \cdot 18x$

$c'(x)$ is positive if $\frac{18x}{9x^2+5} > 0$

observe that denominator is always positive so

we want $18x > 0$

$\underline{\underline{x > 0}}$ or $(0, \infty)$

III) $p = -.2x + 16$

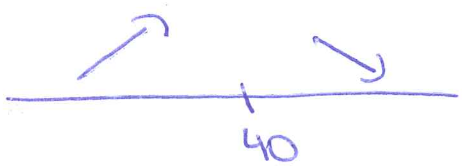
$R(x) = p \cdot x = (-.2x + 16)x = -.2x^2 + 16x$

$R'(x) = -.4x + 16$

$0 = -.4x + 16$

$+.4x = 16$

$x = 40$



so $\underline{x = 40}$ is relative max but b/c it is the only critical pt it must be absolute max as well

IV) $C(x) = 5 + 8x$

$p = -.2x + 16$

$P(x) = R(x) - C(x)$

$= (-.2x^2 + 16x) - (5 + 8x)$

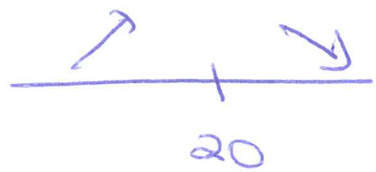
$= -.2x^2 + 8x - 5$

$P'(x) = -.4x + 8$

$0 = -.4x + 8$

$.4x = 8$

$x = 20$



$\underline{\underline{x = 20 \text{ is abs. max}}}$