MATH 1071Q

Exam 02 review Solutions

(01)

i) $f(x) = \sqrt{2}$ $\quad f'(x) = 0$

ii) $f(x) = x^{1.4}$ $\quad f'(x) = 1.4x^{1.4-1} = 1.4x^{0.4}$

iii) $f(x) = 4 - x - x^2$ $\quad f'(x) = -1 - 2x$

iv) $f(x) = \frac{1}{3}e^x$ $\quad f'(x) = \frac{1}{3}e^x$

v) $f(x) = 4 - 3\ln x$ $\quad f'(x) = -3 \cdot \frac{1}{x} = \frac{3}{x}$

vi) $f(x) = x^5 - 5e^x - 1$ $\quad f'(x) = 5x^4 - 5 \cdot e^x = 5(x^4 - e^x)$

vii) $f(x) = \dfrac{x^2 - 3x - 6}{3x}$

$$f'(x) = \frac{3x(2x-3) - 3(x^2-3x-6)}{(3x)^2} = \frac{6x^2 - 9x - 3x^2 + 9x + 18}{9x^2}$$

$$= \frac{3x^2+18}{9x^2} = \frac{3(x^2+6)}{3 \cdot 3x^2} = \frac{(x^2+6)}{3x^2}$$

viii) $f(x) = 3x^2 + \dfrac{3}{x^2} \Rightarrow f(x) = 3x^2 + 3x^{-2}$

$$f'(x) = 3(2x) + 3(-2x^{-3}) = 6x - 6x^{-3} = 6(x - x^{-3})$$

ix) $f(x) = \dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} \Rightarrow f(x) = x^{-1} + x^{-2} + x^{-3}$

$$f'(x) = -x^{-2} - 2x^{-3} - 3x^{-4}$$

x) $f(x) = \dfrac{x + 3/x}{\sqrt{x}} \Rightarrow f(x) = \left(x + \dfrac{3}{x}\right) \div x^{1/2}$

$f(x) = \left(x + \dfrac{3}{x}\right) \cdot \dfrac{1}{x^{1/2}} \Rightarrow f(x) = \underline{\dfrac{x}{x^{1/2}} + \dfrac{3}{x^{3/2}}}$

$f(x) = x^{1/2} + 3x^{-3/2} \Rightarrow f'(x) = \dfrac{1}{2}x^{-1/2} + 3\left(-\dfrac{3}{2}x^{-5/2}\right)$

$\underline{f'(x) = \dfrac{1}{2}x^{-1/2} - \dfrac{9}{2}x^{-5/2}}$

xi) $f(x) = \sqrt{x^3} \Rightarrow f(x) = (x^3)^{1/2} \Rightarrow \underline{f(x) = x^{3/2}}$

$f'(x) = \dfrac{3}{2}x^{1/2}$

xii) $f(x) = x^3 \cdot e^x$

$f'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = \underline{x^2 e^x (3+x)}$

xiii) $f(x) = x^4 \ln x$

$f'(x) = 4x^3 \ln x + x^4 \cdot \dfrac{1}{x} = 4x^3 \ln x + x^3 = \underline{x^3(4\ln x + 1)}$

xiv) $f(x) = (e^x + 1)(\sqrt{x} + 1) \Rightarrow f(x) = (e^x + 1)(x^{1/2} + 1)$

$f'(x) = e^x \cdot (x^{1/2} + 1) + (e^x + 1) \cdot \dfrac{1}{2}x^{-1/2}$

xv) $f(x)=\left(e^x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right) \Rightarrow f(x)=\left(e^x+x^{-1}\right)\left(1+x^{-2}\right)$

$f'(x)=\left(e^x-x^{-2}\right)\left(1+x^{-2}\right)+\left(e^x+x^{-1}\right)\left(-2x^{-3}\right)$

xvi) $f(x)=\frac{x}{x+2}$

$f'(x)=\frac{(x+2)\cdot(1)-x(1)}{(x+2)^2}=\frac{x+2-x}{(x+2)^2}=\underline{\underline{\frac{2}{(x+2)^2}}}$

xvii) $f(x)=\frac{x+1}{x^2+2}$

$f'(x)=\frac{(x^2+2)\cdot 1-(x+1)\cdot 2x}{(x^2+2)^2} \Rightarrow f'(x)=\frac{x^2+2-2x^2-2x}{(x^2+2)^2}$

$f'(x)=\frac{2-x^2-2x}{(x^2+2)^2}$

xviii) $f(x)=\frac{e^x}{x-2}$

$f'(x)=\frac{(x-2)\cdot e^x-e^x(1)}{(x-2)^2} \Rightarrow f'(x)=\frac{e^x(x-2-1)}{(x-2)^2}=\frac{e^x(x-3)}{(x-2)^2}$

xix $f(x)=\frac{-4\ln x}{x^4+3}$

$f'(x)=\frac{(x^4+3)\left(-4\cdot\frac{1}{x}\right)-(-4\ln x)(4x^3)}{(x^4+3)^2}$

$f'(x)=\frac{-(x^4+3)\cdot 4/x+16\ln x\cdot x^3}{(x^4+3)^2}$

XX) $f(x) = e^{3x} \Rightarrow f(x) = e^x \cdot e^x \cdot e^x$

$f'(x) = e^x \cdot (e^x \cdot e^x) + e^x \cdot (e^x \cdot e^x)'$

$f'(x) = e^x \cdot e^x \cdot e^x + e^x [e^x \cdot e^x + e^x \cdot e^x]$

$= e^x \cdot e^x e^x + e^x e^x e^x + e^x e^x e^x$

$= 3e^{3x}$.

(XXI) $f(x) = e^{3x}$

$f'(x) = e^{3x} \cdot 3 = \underline{\underline{3e^{3x}}}$.

XXII) $f(x) = (2x+1)^{15}$

$f'(x) = 15(2x+1)^{14} \cdot 2 \Rightarrow f'(x) = 30(2x+1)^{14}$

XXIII) $f(x) = \sqrt{e^x} \Rightarrow f(x) = (e^x)^{1/2} \Rightarrow f(x) = e^{x/2}$

$f'(x) = e^{x/2} \cdot \frac{1}{2} \Rightarrow f'(x) = \frac{1}{2} e^{x/2}$

XXIV) $f(x) = \frac{3}{e^x+1}$

$f'(x) = \frac{(e^x+1) \cdot 0 - 3e^x}{(e^x+1)^2}$

$f'(x) = \frac{-3e^x}{(e^x+1)^2}$

or $f(x) = 3(e^x+1)^{-1}$

$f'(x) = 3 \cdot -1 (e^x+1)^{-2} \cdot e^x$

$f'(x) = -3e^x (e^x+1)^{-2}$

$f'(x) = \frac{-3e^x}{(e^x+1)^2}$

XXV) $f(x) = \dfrac{1}{\sqrt{\ln x}} \Rightarrow f(x) = \dfrac{1}{(\ln x)^{1/2}} \Rightarrow f(x) = (\ln x)^{-1/2}$

$f'(x) = -\dfrac{1}{2}(\ln x)^{-3/2} \cdot \dfrac{1}{x} \Rightarrow f'(x) = -\dfrac{1}{2x} \cdot (\ln x)^{-3/2}$

$f'(x) = \dfrac{-1}{2x(\ln x)^{3/2}}$

XXVI) $f(x) = e^x (x^2+1)^8$

$f'(x) = e^x (x^2+1)^8 + e^x \cdot 8(x^2+1)^7 \cdot 2x$

$= e^x (x^2+1)^7 (x^2+1+16x)$

XXVII) $f(x) = (5x+3)^3 (x^2-4)^6$

$f'(x) = 3(5x+3)^2 \cdot 5 (x^2-4)^6 + (5x+3)^3 \cdot 6(x^2-4)^5 \cdot 2x$

$f'(x) = (5x+3)^2 (x^2-4)^5 (15(x^2-4) + 12x(5x+3))$

XXVIII) $f(x) = \sqrt{\sqrt{x}+1} \Rightarrow f(x) = (x^{1/2}+1)^{1/2}$

$f'(x) = \dfrac{1}{2}(x^{1/2}+1)^{-1/2} \cdot \dfrac{1}{2} x^{-1/2}$

$f'(x) = \dfrac{1}{4} \dfrac{1}{(x^{1/2}+1)^{\frac{1}{2}} \cdot x^{1/2}}$

XXIX) $f(x) = \dfrac{e^x + 1}{(2x+3)^3}$

$$f'(x) = \frac{(2x+3)^3 \cdot e^x - (e^x+1)\cdot 3(2x+3)^2 \cdot 2}{\left((2x+3)^3\right)^2}$$

$$f'(x) = \frac{(2x+3)^2\left[(2x+3)e^x - 6(e^x+1)\right]}{(2x+3)^6}$$

$$f'(x) = \frac{(2x+3)e^x - 6(e^x+1)}{(2x+3)^4}$$

XXX) $f(x) = e^{\sqrt[3]{x}} \Rightarrow f(x) = e^{x^{1/3}}$

$$f'(x) = e^{x^{1/3}} \cdot \frac{1}{3}x^{-2/3}$$

XXXI) $f(x) = \ln|x^3+x^2+1|$

$$f'(x) = \frac{1}{x^3+x^2+1}\cdot(3x^2+2x)$$

XXXII) $f(x) = \ln\left|\frac{x}{x+1}\right|$

$$f'(x) = \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)\cdot 1 - x\cdot 1}{(x+1)^2}$$

$$f'(x) = \frac{x+1}{x} \cdot \frac{x+1-x}{(x+1)^2} \Rightarrow f'(x) = \frac{x+1}{x} \cdot \frac{1}{(x+1)^2} = \underline{\underline{\frac{1}{x(x+1)}}}$$

XXXIII) $f(x) = e^{-x^2}\ln(x^2)$

$$f'(x) = -2x\,e^{-x^2}\ln(x^2) + e^{-x^2}\frac{1}{x^2}\cdot 2x$$